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RESEARCHES ON THE ROTATION OF PERMALLOY AND SOFT IRON BY MAGNETIZATION AND THE NATURE OF THE ELEMENTARY MAGNET.

BY S. J. BARNETT.

RESEARCHES ON THE ROTATION OF PERMALLOY AND SOFT IRON BY MAGNETIZATION AND THE NATURE OF THE ELEMENTARY MAGNET.[1]

By S. J. Barnett

§ 1. *The gyromagnetic ratio.* The most important quantities associated with the elementary magnet in ferromagnetic substances are its magnetic moment μ_0, its angular momentum M_0, and its gyromagnetic or magneto-mechanical ratio $\rho = M_0/\mu_0$. The gyromagnetic ratio of an elementary magnet consisting of a Weber-Rutherford-Bohr magneton, or electron moving in an orbit about a nucleus or atomic core, is $\rho_B = 2m/e$; that of a Lorentz electron spinning on a diameter is $\rho_L = m/e = \frac{1}{2} \rho_B$; that of an Abraham electron, uniformly charged throughout its volume and spinning about a diameter, is $\rho_A = 6/7.\ m/e$; the quantity m/e being the ratio of electron mass to electron charge. Although the magnetic moments of many free atoms have now been determined by the method of Gerlach and Stern, no method of determining M_0 or μ_0 separately and with certainty in magnetic aggregates has yet been devised; but in several investigations $\rho = M_0/\mu_0$ has now been determined with considerable precision for many ferromagnetic substances.

§ 2. *Magnetization by rotation. The gyromagnetic anomaly.* The first successful work in the domain of gyromagnetic phenomena was an investigation on the magnetization of iron by rotation[2] first published by me in 1914 by presentation to the Ohio Academy of Sciences and the American Physical Society. This work established for the first time the actual existence of Ampère's molecular currents, proved that the electricity in motion in them is negative, and furnished data from which it is possible to calculate $\rho = M_0/\mu_0$ for the magnetic element in iron. The value of ρ calculated from these data and similar data obtained a few months later by the same general method is equal, within less than the experimental error of some 10 or 15 per cent, to m/e ($2m/e = 1.13 \times 10^{-7}$ e.m.u), and thus to one-half the gyromagnetic ratio for an electron orbit—a result which has come to be known as the *gyromagnetic* or *magneto-mechanical anomaly.*

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[1] Reports of progress on this work presented to the American Physical Society have been published in abstracts as follows: Physical Review 30, p. 964, 1927; *ibid.*, 31, p. 1116, 1928; *ibid.*, 36, p. 789, 1930.

[2] S. J. Barnett, Phys. Rev. 6, p. 239, 1915.

This work on magnetization by rotation was later (1917–1924) extended to other ferromagnetic substances, the last paper[3] dealing with iron, nickel, cobalt, Heusler alloy, and iron-nickel, iron-cobalt, and cobalt-nickel alloys. The mean gyromagnetic ratio obtained for all these substances was $1.06 \times m/e$, with an error estimated as

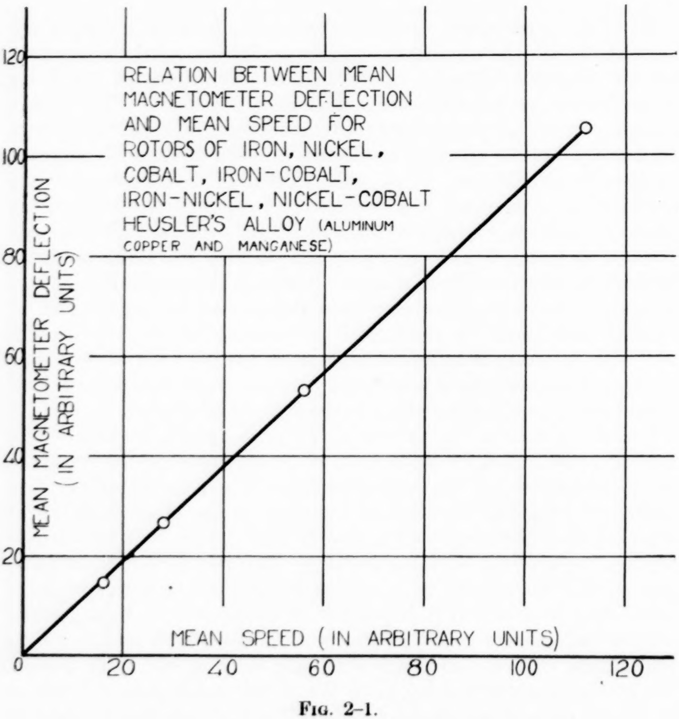

Fig. 2–1.

not greater than 2 per cent, and with no certain evidence of any difference between the gyromagnetic ratios of the elementary magnets in different substances. The theory of the experiments requires that the change in the magnetic moment of the body, and the deflection

[3] S. J. Barnett and L. J. H. Barnett, Proc. Amer. Acad. 60, p. 127, 1925.

of the measuring instrument produced thereby, be proportional to the speed of rotation. The experimental relation between speed and reduced deflection for the last and most extensive series of observations on all the above mentioned substances is given in Fig. 2–1, from which an idea of the degree of precision of the work can be obtained. The reduced deflection for each rotor is the actual deflection divided by the change of moment produced by a standard weak axial magnetic field intensity.

§ 3. *Rotation by magnetization.* The converse phenomenon, viz., the rotation of a ferromagnetic substance by magnetizing it, was discovered in 1915 by Einstein and de Haas,[4] who were not then acquainted with the (earlier) work on magnetization by rotation. Instead of m/e, their work, upon iron, gave for ρ the value $2m/e = \rho_B$, which they estimated correct within about 10 per cent; but additional investigations,[5] made much later in the same year and in the next year, were necessary to give them any certain evidence as to the sign of the electricity involved. This later work showed that the electricity was negative, as had been already definitely proved in the work on magnetization by rotation.

The work of Einstein and de Haas was limited to a few observations on iron. A more extensive and thorough investigation, on both iron and nickel, was made by John Q. Stewart[6] in this country in 1918. He obtained for both substances a value of ρ half that obtained by Einstein and de Haas, and thus equal to that obtained in the converse experiments several years earlier.

This work has been followed and confirmed by several other excellent investigations, in Switzerland, Sweden, and England. Of this work probably the most precise has been done in England, where Chattock, Sucksmith, and Bates[7] have made studies of iron, nickel, cobalt, magnetite and Heusler's alloy. In every case they believe they have, like all others except Einstein and de Haas, found for ρ the value m/e within their experimental error. In their best work, which was done on iron and nickel, they believe the experimental error not greater than one per cent. There is thus a discrepancy of about 6%

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[4] A. Einstein and W. J. de Haas, Verh. d. D. Phys. Ges. 17, p. 152, 1915.

[5] W. J. de Haas, Verh. d. D. Phys. Ges. 18, p. 423, 1916; A. Einstein, ibid. 18, p. 173, 1916.

[6] J. Q. Stewart, Phys. Rev. 11, p. 100, 1918.

[7] Phil. Trans. Roy. Soc. A, 223, p. 257, 1922; Proc. Roy. Soc. A, 104, p. 499, 1923; Proc. Roy. Soc. A, 108, p. 638, 1925.

between their mean result and the value obtained from experiments on magnetization by rotation.[8]

§ 4. *The present investigation.* On account of all the discrepancies, and the great theoretical importance of the phenomena, I have for several years been making a new and elaborate investigation of the rotation of certain substances by magnetization. An account of the investigation is given in this paper.

§ 5. *General experimental methods and theory.* In all experiments on the Einstein and de Haas phenomenon a cylinder of the substance under investigation is suspended with its axis vertical from a fixed support by means of a vertical wire or fiber along the axis extended. The cylinder is generally mounted coaxially within a magnetizing coil of insulated wire, and the motion of the cylinder produced by changes of axial magnetization is studied.

Let μ_0, as above, denote the magnetic moment of one elementary magnet, M_0 its angular momentum, and ρ its gyromagnetic ratio, so that

$$M_0 = \rho\mu_0 \tag{5-1}$$

Let j' denote the resultant angular momentum of all the elementary magnets in the cylinder, or *rotor*, as it will be called, and μ the rotor's axial magnetic moment. Then

$$j' = \Sigma M_0 \cos \theta = \rho\Sigma\mu_0 \cos \theta = \rho\mu$$

where θ is the angle made by the axis of an elementary magnet with the geometrical axis of the rotor. When the magnetization changes, the total angular momentum of all the elementary magnets will change at the rate dj'/dt; so that, in accordance with the principle of the conservation of momentum, the angular momentum of the *rotor and solenoid together* will change at the rate

$$g = dj/dt = - dj'/dt = - \rho d\mu/dt \tag{5-2}$$

which is the *gyromagnetic torque* on the rotor and solenoid.

[8] The discrepancy is very probably due mostly, if not entirely, to the failure of the British physicists to eliminate the inportant errors discussed below in § 9 (2), § 11 (c), § 12 (b), § 17, and § 21, which were not considered by them or by any earlier investigators, but which are usually of considerable magnitude, and sometimes very large.

In ballistic methods of investigation equation (5–2) is used in the modified form

$$\int g dt = - \rho \Delta \mu \tag{5–3}$$

i. e., the angular impulse is determined for a given change $\Delta\mu$ in the magnetic moment. In alternating current methods equation (5–2) is used directly, always with more or less complete resonance in order to obtain sensibility and to eliminate certain errors.

In nearly all the experimental work done hitherto the coil magnetizing the rotor has been fixed to the earth, and only the momentum communicated to the *rotor* has been measured, whereas the theory gives only the combined torque upon *rotor and solenoid.*[9] In the investigations by de Haas, however, the magnetizing coil was rigidly wound on the rotor, so that the total torque was measured; but this was done for practical, not theoretical, reasons; and the precision of the work was so low that no conclusion can be drawn from it with regard to the effect of the rigid connection between coil and rotor.

In the investigation described here many precise experiments have been made with the magnetizing coil wound rigidly upon the rotor, and also many with the coil fixed to the earth; and it has been definitely proved that the reaction to the electron change of momentum is at least almost entirely upon the rotor.

§ 6. *Alternating current null method.* All the work described here has been done by alternating current methods. The method to be described first, but not the principal method used, is a null method, and resembles, but with great differences, a method used by Sucksmith and Bates, and suggested by Chattock. The torque given to the rotor by changing its magnetization has its effect in producing rotation annulled by another torque, as suggested by Chattock. This annulling torque is produced by the action of an electric current traversing a fixed coil of wire, which exerts a torque on a small permanent magnet rigidly attached to the rotor. A diagram of some of the chief parts of the apparatus is given in Fig. 6–1.

The roctor, *F*, is axially suspended by a thin German silver wire or strip, *D*, attached above to a brass rod, *C*, passing through a torsion head, *B*, of amber. Hanging from the lower end of the rotor by an essentially rigid joint is a small brass rod, *J*, carrying on opposite

[9] Attention was directed to the remote possibility of error from this source by O. W. Richardson in Phys. Rev. 26, p. 248, 1908.

sides a pair of parallel mirrors, *I*, and below, a group, *L*, of small permanent magnets, all turned in the same direction. Hanging from

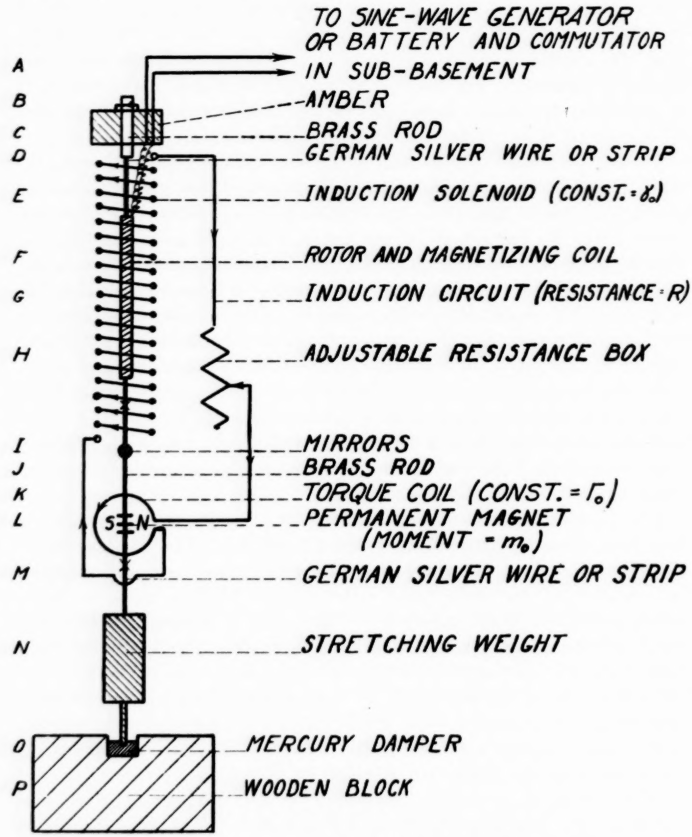

FIG. 6–1. Diagram illustrating chief experimental methods.

the lower end of the brass rod, *J*, by another German silver suspension, *M*, is a stretching weight, *N*, which keeps the axes of the rods in line and the suspensions tight. A small rod projecting downward from

the weight dips into some mercury, O, in a wooden cup, P, and helps to dampen out any lateral motion of the hanging bodies.

In the diagram the magnetizing coil is shown wound directly upon the rotor, F. Current is led to and from the coil by two fine wires soldered to small brass rods passing through the torsion head and connected by the wires A to an alternating current generator, or to a storage battery through a rotating commutator, driven by a motor in the subbasement. In the diagram, for the sake of clearness, the two fine wires forming the ends of the magnetizing coil are shown coiled separately. In the actual apparatus they were twisted together, and then the double wire twisted into a loose spiral around the suspension D. In another form of the apparatus the rotor is magnetized by a fixed solenoid located inside the induction solenoid referred to below. Most of the work was done with commutators and storage battery, not with the sine-wave generator, chiefly in order to make the phase angles α, γ, δ, § § 10–13, small.

When the commutator or generator is in operation the rotor is acted on by an alternating torque. In order to make the rotary vibration of the rod thus produced as great as possible, the speed of the commutator (or generator), controlled by devices described below, is adjusted until approximate resonance occurs, when the amplitude of vibration is very many times as great as it would be from a single impulse. The range, or double amplitude, of the vibration is examined by lamp, mirror, lens and scale.

The rotor is surrounded with a much longer and uniformly wound cylindrical coil of wire, E, which will be called the *induction solenoid;* and the group, L, of small permanent magnets is at the center of a small double coil, K, the *torque coil,* the planes of whose turns are parallel to the magnets. Currents sent in opposite directions through this coil will produce opposite torques upon the magnets and thus upon the vibrating system; so that, if such currents are properly timed, and of the proper magnitude, they will just annul the gyromagnetic torque.

This condition is brought about as follows: The induction solenoid E is connected through an adjustable resistance box H in circuit with the torque coil K. When the magnetization of the rotor is changing it induces a current in this circuit, whose magnitude can be increased or decreased at will by changing the resistance in the box. Thus, at the same time as a gyromagnetic torque is given to the rotor system, a torque of electromagnetic origin, which may be made exactly equal

to it, is also given to the system. If the terminals of the torque coil are connected in one way to the rest of the circuit, the two torques will be in the same direction and the resultant torque twice one of them; if the connections are reversed the two torques will annul one another, and the vibration will disappear. From the directions in which the coils are wound, and the permanent magnet turned, it is easy to predict in advance how the terminals should be connected to produce the compensation, provided we know whether the electricity in Ampère's whirls is negative or positive, the connections being opposite for the two signs. The connections found necessary always show that the *electricity is negative,* and are correctly indicated in the diagram for the given azimuth of the magnet L.

If R_0 is the resistance of the circuit when the vibration is annulled, m_0 the moment of the group of permanent magnets, and Γ_0 and γ_0 the *constants* of the torque coil and induction solenoid, it is easy to show that, with due attention to signs (see § 7),

$$-\rho = \frac{\Gamma_0 \gamma_0 m_0}{R_0} \tag{6–1}$$

provided that the two effects we have been considering are the only ones involved.

§ 7. *Conventions with respect to signs of torques etc.* In what follows all directed quantities associated with the diagram of Fig. 6–1 which can be represented by vertical lines in a plane parallel to that figure will be considered positive when directed upward; and the right handed screw convention will be adopted. Thus an axial torque which would move the ends N and S of the magnet L from and to the reader, respectively, would be *positive*. The current in the torque coil K will be considered positive when it is clockwise. Horizontal quantities will be considered positive when directed to the right or when directed from the reader.

§ 8. *Formula for null method in ideal case.* To develop the formula (6–1) it will be necessary to take from § 5 the relation

$$g = -\rho d\mu/dt \tag{8–1}$$

which is true whatever the form of the wave of electromotive force or magnetic moment.

Let ϕ denote the magnetic flux through the induction circuit produced by μ, and suppose the flux produced by the magnetizing

coil negligible. Then, if the constant γ_0 of the induction solenoid is uniform throughout the volume occupied by the rotor,

$$\phi = \mu\gamma_0 \tag{8–2}$$

If, as will be assumed in accordance with the facts, the reactances of the induction circuit for all appreciable harmonics are negligible in comparison with its resistance R, the current q produced in this circuit by the changing flux will be

$$q = -\frac{1}{R}\frac{d\phi}{dt} = -\frac{\gamma_0}{R}\frac{d\mu}{dt} \tag{8–3}$$

and will thus have the same form as g.

Thus the torque on the vibrator due to the action of the field of the torque coil upon its permanent magnet, with moment m_0, will be

$$c = \Gamma_0 m_0 q = -\frac{\Gamma_0\gamma_0 m_0}{R}\frac{d\mu}{dt} \tag{8–4}$$

with the same form as that of g.

If R is given such a value R_0, and the connections between the torque coil and the induction solenoid are so made, that $c = -g$ the resultant torque will vanish at every instant, and we shall thus have

$$-\rho = \frac{\Gamma_0\gamma_0 m_0}{R_0} \tag{8–5}$$

which is identical with (6–1)

§ 9. *Sources of disturbance.* Actual experimental conditions are of course never so nearly ideal as they are assumed to be in § § 6 and 8. There are in general many effects which tend to mask the gyromagnetic effect being studied, and whose elimination, by processes described later, is the source of most of the difficulties involved in the work. Some of these effects are as follows:

(1) *Mechanical vibrations* due to the continual shaking of the building. These can be largely eliminated by hanging the apparatus from the ceiling by springs and damping the motion with cotton waste,[10] and by working as far as possible when the earth's tremors are

[10] *Note added in proof.*—Since this was written the behavior of the apparatus with respect to vibrations has been greatly improved by discarding the cotton waste (sometimes removed before) and applying adhesive tape along the springs and from spring to spring and by adopting also the method of internal damping suggested by R. Müller (Ann. des Phys. 1, p. 613, 1929). This con-

slight. The motor, gears, and generator, in the subbasement, never produced any appreciable vibration, either rotary or other, of the rotor.

(2) *Magnetostriction*, or changes in the rotor's dimensions due to changes in magnetization. The effects are relatively small in permalloy and soft iron, and much worse in other substances. Even in permalloy they may produce errors of many per cent. Some of them appear to be reduced by sealing the parts of the suspended system together with shellac or duco cement (which is better), and by changing the suspended weight. A serious systematic error due jointly to magnetostriction, inequality of the two half-cycles of magnetizing current, and either asymmetry of the suspended system or the variation with tension of the twist of the suspension, or both, can be eliminated by periodically reversing the connections between the magnetizing coil and the source of power. If an alternating current generator is used the same result may be accomplished by reversing its field. The effect can be reduced in some cases by using magnetizing currents strong enough to produce approximate saturation of the magnetic material. See also § 45.

Even when the half-cycles of current are equal, a similar magnetostrictive torque may exist if the rotor has a sufficient permanent vertical moment, due to earlier vertical magnetization or to failure to compensate the vertical part of the intensity of the earth's field. In the latter case the effects can be eliminated by neutralizing or reversing this part of the field. With certain rotors this effect is very pronounced when the current is weak and the residual vertical field not entirely compensated. In the former case the behavior of the rotor can be improved by putting it through a demagnetizing process.

(3) *Leakage* between the magnetizing circuit and the induction circuit. The effect of this can be made negligible by taking proper precautions with respect to insulation.

(4) *The presence of the earth's magnetic field*, which acts on the rotor and rotor coil. This can be largely compensated by means of three coils carrying steady electric currents, one to compensate the vertical part of the earth's intensity Z, one to compensate the northerly

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sists simply in placing pans of oil on the suspended apparatus. For referring me to Müller's work I am indebted to Dr. John Strong. As a further improvement, made possible by the changes already mentioned, small vanes were placed on the German silver rod of Fig. 6–1, and the mercury was replaced by a reservoir of light oil into which the weight N dipped.

component X, and one to compensate the easterly component Y. These compensations are more troublesome than they would otherwise be because the earth's intensity is continually varying in both magnitude and direction, and because the work has to be done in a building which contains iron, interfering with the uniformity of the field.

(5) *Failure,* in the rotor and other parts of the vibrating system, *of complete symmetry* about a central vertical line. To make the system sufficiently symmetrical mechanically requires the greatest care and skill on the part of the instrument maker. But even a mechanically perfect system is of course in general not magnetically so. The magnetic asymmetry may be made small with less difficulty in the case of easily magnetizable substances like soft iron and permalloy, than in the cases of at least most others.

(6) *Magnetic actions,* in addition to those given above, *of the different electric coils* on the rotor and the small group of permanent magnets. These can be made negligible or eliminated as indicated below.

(7) *Electron-inertia,* on account of which the change of current in the magnetizing coil fixed on the rotor, or the change of magnetic flux through the rotor, inducing currents therein, produces an axial torque. This is very minute, but not negligible in all cases. When not negligible, it can be computed and allowed for.

(8) *Inductive action* of the current in the magnetizing coil on the secondary or induction circuit. This can be computed and allowed for, or can be eliminated by the use of extra compensating coils.

(9) *The Elihu Thomson electrodynamic repulsion effect.* This is not in general appreciable. If it were appreciable in the case of equal half-cycles of current, it would have a negligible effect in the resonance experiments, as its frequency would be twice that of the vibration. In the case of unequal half-cycles there would be a component of the same frequency as that of the vibration, which could be eliminated by the process of reversal mentioned in discussing (2), above.

(10) *Magnetic impurities* in critical parts of the testing magnetometers, torque coil, induction solenoids, and other apparatus assumed to be free from iron. In this work careful tests of all material of importance have shown that no appreciable error could result from such impurities as were present.

In what follows all the torques which can produce appreciable

systematic effects and have not already been adequately treated, as well as some others, are discussed in sufficient detail.

§ 10. *The normal torques.* The frequency of the generator or commutator being $\omega/2\pi$, the first harmonics of the vertical and horizontal magnetic moments of the rotor will be assumed to be

$$\mu = M \sin \omega t \quad \text{and} \quad \nu = N \sin (\omega t - \alpha) \qquad (10\text{–}1)$$

respectively; and the angular momentum of the complete vibrating system will, on account of resonance, be assumed to be

$$p = P \sin (\omega t + \beta) = KA \sin (\omega t + \beta) \qquad (10\text{–}2)$$

where A is the amplitude of the strip of light on the scale and K is a constant. Then the torques acting upon the vibrating system, aside from the disturbing torques, will be as follows:

(1) *The total torque* τ. This will be equal to the rate change of angular momentum. Thus

$$\tau = \frac{dp}{dt} = \omega P \cos (\omega t + \beta) = \omega KA \cos (\omega t + \beta)$$
$$= T \cos (\omega t + \beta) \qquad (10\text{–}3)$$

(2) *The gyromagnetic torque g.* From (8–1) and (10–1) this torque is evidently

$$g = -\rho\omega M \cos \omega t = G \cos \omega t \qquad (10\text{–}4)$$

(3) *The torque c* due to a current $q = Q \cos \omega t$ in the torque coil. As above,

$$c = \Gamma_0 m_0 q = \Gamma_0 m_0 Q \cos \omega t = C \cos \omega t \qquad (10\text{–}5)$$

(4) *The frictional torque f.* This will be given by the equation

$$f = -\sigma p = -\sigma KA \sin (\omega t + \beta) = \sigma KA \sin (\omega t + \beta + \pi)$$
$$= F \sin (\omega t + \beta + \pi) \qquad (10\text{–}6)$$

where σ is a positive constant.

(5) *The suspension torque.* This torque, in phase with τ, will be

$$s = S \cos (\omega t + \beta) = \omega KA \frac{\omega_0^2}{\omega^2} \cos (\omega t + \beta) \qquad (10\text{–}7)$$

where $\omega_0/2\pi$ is the natural frequency of the vibration.

§ 11. *The disturbing torques.* (1) *Magnetic torques.* Undoubtedly the most important of the disturbing torques to be eliminated are the *magnetic torques.* These are due either to the residual part of the earth's field (A), or to the action of the electric coils on the movable system (B).

(A) *Torques due to the earth's field.* Let ΔH denote the horizontal intensity of the residual magnetic field in which the rotor is placed, and ΔX and ΔY its northerly and easterly components; also let ΔZ denote the vertical intensity of the residual field.

(a) Then there will be an axial torque on the rotor due to ΔH, which may be written

$$e_h = h\Delta H\nu = E_h \sin (\omega t - \alpha) \tag{11-1}$$

If the torsion head is turned through 180° this torque will be reversed and will become

$$e'_h = - e_h = - E_h \sin (\omega t - \alpha) \tag{11-2}$$

E_h of course vanishes with ΔH, ν, and the angle between them.

(b) ΔH, acting on the vertical moment, will produce a torque

$$l = L \sin \omega t \tag{11-3}$$

about a horizontal axis. The natural frequency n of vibration about such an axis is clearly very much greater than the natural axial frequency $\omega_0/2\pi$, with which at least approximate resonance is always maintained, and the damping is great. Thus the resulting amplitude in altitude is very minute. If the system is not axially symmetrical, however, the torque will, in general, give rise to an axial torque

$$b = B \sin (\omega t - \chi) \tag{11-4}$$

and thus to axial oscillations. This torque changes sign when the torsion head is turned through 180°. In addition to the axial oscillations due to this torque, a group of minute free axial vibrations with the frequency n will be set up during each half cycle of the magnetization.

All these effects may be made very minute, and vanish with ΔH.

(c) Although no earlier investigator has considered it necessary to annul Z, the residual intensity ΔZ, acting on the horizontal moment ν, may produce effects quite similar to those just considered, and for the same reason. The effective axial torque may be written

$$e_z = E_z \sin(\omega t - \alpha) \qquad (11\text{–}5)$$

It does not change sign or magnitude when the torsion head is turned in azimuth, but does change sign with ΔZ and E_z. In the cases of the permalloy and soft iron rotors this torque, as would be expected from the small value of ν, is very small even when ΔZ is a considerable part of the vertical intensity of the earth's field. As examples, all of them complicated however with effects of magnetostriction, two series of experiments on the iron rotor $I^2{}_{2\cdot4-2}$ may be mentioned, together with one on a permalloy rotor.

With 50 milliamperes traversing the magnetizing coil on the rotor, and only approximate resonance, the mean amplitude with the earth's vertical intensity compensated was 13.22 cm; with the earth's intensity alone, 13.20 cm; with the earth's intensity doubled, 13.16 cm. The same night, with 100 milliamperes, the three arrangements gave, respectively, 11.30 cm, 11.05 cm, and 11.29 cm.

With the same rotor a number of complete sets of observations have been taken by the latest method with the earth's vertical intensity under and over compensated by equal amounts and have shown a variation in the calculated value of ρ of only about 4.4 % for a change of more than 20% in the vertical intensity.

In the case of the permalloy rotor ${}_sP_{2\cdot4-2}$ changes in the vertical intensity equal to more than 12% of the earth's vertical intensity made no appreciable effect on the value of ρ.

(d) If the magnetizing coil is wound upon the rotor, there will be a torque upon the system due to the action of the residual field upon the wire, which may be written

$$i = I \sin(\omega t + \gamma) \qquad (11\text{–}6)$$

where γ is very small for rectangular waves. This torque retains its magnitude but changes its sign when the torsion head is turned through 180°.

§ 12. (B) *Torques due to the action of the electric coils on the moveable system.* (a) If the magnetizing coil is fixed to the earth, and if the rotor has a permanent horizontal magnetic moment ξ, there will, in general, be an axial torque d upon the rotor due to the alternating field of the fixed coil, whose direction is not, in general, exactly normal to ξ. This torque, with the commutator producing the alternating current, will be nearly in phase with μ. Thus we may write

$$d = D \sin(\omega t + \delta) \qquad (12\text{–}1)$$

where δ is small. This torque also will evidently be reversed in sign by turning the torsion head through 180°. It is often of very considerable magnitude, and may be difficult to eliminate completely. When rectangular waves are used D can probably be reduced by inserting a suitable condenser in the magnetizing circuit—a device used with effect by Einstein (Footnote 5 § 2) in the ballistic method.

(b) In this same case there is an axial torque due to the action of the alternating field on the alternating horizontal moment ν; but if the half-cycles of current and magnetization are equal, its fundamental frequency is twice that of the current, and so its effect is negligible. If the half cycles are not equal, there may be also a residual torque, with the frequency of the alternating field, which may have any phase relation to that of g. This torque may be written

$$j = J \sin (\omega t + \zeta) \tag{12–8}$$

So far as the half-cycles of current are concerned, it can be eliminated by making the reversal referred to in § 9, (2).

(c) The torque coil produces a magnetic field extending upward into the region occupied by the rotor, where its lines of intensity are horizontal. Thus, if the rotor's magnetization is not symmetrical about the axis of suspension, an axial torque may be produced. If the magnetization is strictly alternating (and half-cycles equal), this torque will have twice the frequency of the alternation, and may be left out of account in resonance experiments. If there is a residual permanent moment, however, produced by an uncompensated part of the earth's field or otherwise, there will be an axial torque of the frequency of the current and in phase with (or in opposition to) the gyromagnetic torque. In the work described here this torque was entirely negligible.

Thus at a point 20 cm distant from the center of the compensating coil (less than the distance between the lower end of rotor and center of coils) and on a line normal to the axis and passing through the center, the magnetic intensity was found to be only one thousandth that at the center.

If now the iron rotor $I^2{}_{2\cdot 4-2}$ (a rotor with one of the largest moments), were permanently magnetized to the maximum magnetization attained in the experiments, and were hung so unsymmetrically as to have the center of its lower end displaced one-half mm. from the vertical line through the center of the upper end, it is easy to show that the axial torque upon it, due to this field intensity, could

not be greater than about 3×10^{-3} that upon the small permanent magnet at the center—equal to the gyromagnetic torque in the null experiments. The assumptions made are of course extreme, and in the actual experiments the torque could have been only a minute fraction of that calculated above, and was thus entirely inappreciable.

Moreover, in a special series of experiments, this rotor was driven by an alternating current of its own frequency in the torque coil, while a direct current of 100 milliamperes traversed its own solenoid, first in one direction, then in the other, the mean amplitudes for the two directions differed by less than 0.1% of either.

If the torque were appreciable, the effect on the determination of ρ could be eliminated by making two sets of experiments, with rotor azimuths differing by 180° with respect to the axis of the small magnet. This was done in some of the work, but was quite unnecessary so far as the effect under discussion is concerned.

If the half-cycles are unequal, there may be a residual torque with the frequency of the current, whose effect, if appreciable, is eliminated by making the reversals mentioned in § 9, (2).

(d) If the induction solenoid is not exactly vertical, but displaced from the vertical by a small angle α, the intensity of its magnetic field will have a horizontal component $h = \alpha\gamma_0$ per unit current, and may exert torques on the rotor due to its horizontal permanent and alternating moments ξ and ν. The intensity at the bottom of the longest rotors due to the compensating coils is $c = \gamma_0/1000$, approximately, per unit current. Since $\gamma_0/\Gamma_0 = 0.12$, approximately, h/c is approximately 120α. Thus even if α were as great as 1/12 radian (and it was always very much less indeed), the maximum possible axial torque on any permanent horizontal moment of the rotor due to h would be no greater than the maximum possible torque due to c (when the permanent pole off the axis is at the bottom of the rotor).

Even if such a torque were not negligible, its effect on the determination of ρ would be eliminated in the process, almost always adopted, of observing with the rotor in two different azimuths differing by 180°, the connections of the induction solenoid not being reversed with the rotor, like those of the torque coil.

If either the fixed magnetizing coil or the induction solenoid does not extend in both directions considerably beyond the rotor, so that even with the coil axis vertical there is a radial intensity near the ends, the torques due to asymmetry will be much greater than otherwise and much more difficult to eliminate by changing azimuths.

§ 13. C. *Torque due to leak between magnetizing and induction circuits.* If there is such a leak there will be a torque

$$\lambda = \Lambda \sin (\omega t + \gamma) \tag{13-1}$$

on the vibrating system. The sign of the torque will be unchanged when the suspension is turned through 180° and at the same time the connections between compensating coil and induction solenoid are reversed. The coefficient Λ is made very minute by using high insulation,[10] and γ is negligible, at least when the commutator is used. So that the effect is both very minute, and also in quadrature with the gyromagnetic torque.

§ 14. D. *Torques due to mutual induction between magnetizing coil and induction circuit* were always made negligible, or else calculated and the corrections applied, as indicated below.

§ 15. E. *Torques due to electron inertia in rotor coil and rotor.*[11]

(1) *Electron inertia in rotor coil.* When the current in the coil, and therefore the angular momentum of the free electrons, changes, an equal and opposite change occurs in the angular momentum of the rotor. Thus there is an electron inertia torque upon the rotor. If the first harmonic of the current in the coil is $i = I \sin (\omega t + \gamma)$, the the first harmonic of this torque, which is the only one effective on account of resonance, consists of two terms t_1 and t_2 as follows:

$$t_1 = -2\frac{m}{e}\cdot M'\omega \cos (\omega t + \gamma) = T_1 \cos (\omega t + \gamma) \tag{15-1}$$

and

$$t_2 = -2\frac{m}{e}\frac{M'\omega}{I}\cdot \mathit{near}\ \omega\Theta \cos (\omega t + \beta) = -T_2 \cos (\omega t + \beta) \tag{15-2}$$

where M' is the magnetic moment of the coil, a the cross-section of the wire, r the mean radius of the coil, n the number of free electrons per unit volume and Θ the amplitude of the rotor's vibration.

When the commutator is used the angle γ is negligible, so that t_1 is in phase with g. In the null method t_2 vanishes with Θ; in the deflection methods β is nearly $\pi/2$, so that t_2 is in quadrature with g and its effect on the determination of ρ (see § 18) vanishes even when its magnitude is appreciable.

[10] In most of the work the insulation was entirely of amber, hard rubber, paraffin, soft rubber, and sulfur.

[11] See S. J. Barnett, Phys. Rev. 36, p. 786, 1930, and a forthcoming article in the *Philosophical Magazine* Ser. 7, Vol. 11.

For the quantity T_1/G we obtain

$$\frac{T_1}{G} = \frac{2\dfrac{m}{e} \cdot M'\omega}{\rho\omega M} = \text{approx. } 1.9\,\frac{M'}{M} \tag{15–3}$$

a fraction which must be subtracted from the value of ρ obtained without applying the correction. The magnitude of the correction in the experiments described here ranges from about 0.1% to about 2%. For the rotor $I^2{}_{2\cdot 4-2}$ when the current is 100 milliamperes the fractional correction is 0.004. For this rotor we may take, as approximately correct, $r = 0.14$ cm, $a = 0.00005$ cm^2; and we may assume $n = 10^{23}$ per cm^3, and $e = 1.6 \times 10^{-20}$ e.m.u. In standard experiments we may take $\omega = 40$ radians per second, $I = 0.01 \times 4/\pi$ e.m.u., $m = 1270 \times 4/\pi$ e.m.u., $M' = 2.5 \times 4/\pi$ e.m.u. and $\Theta = 0.01$ radian. From these data $T_2/G =$ approximately 0.004.

(2) *Electron inertia in the rotor.* There are also electron-inertia torques upon the rotor due to the induction of currents therein by the changing flux, but it is easy to show that they are very much less even than those just considered. One of the two terms vanishes in the null method; both are in quadrature with g in the deflection method.

§ 16. *Experiments with rotors of brass and glass.* In order to study disturbing torques not due to the rotor's magnetization, rotors of brass and glass were prepared. The electron inertia torque on the rod itself is too minute to be observed for brass, and of course for glass; while the electron-inertia torque on the coil does not depend much on the exactness of its winding. The torques due to the action of the residual magnetic field on the coil, however, depend greatly on the winding, on the arrangement of the leads, or both. In the case of a glass rotor these torques were originally very small, but were greatly increased when the rotor was rewound, although with great care.

In 1827 the following experiments, with others, were made on a brass rotor, which was 0.24 cm in diameter and of the same length as the standard magnetic rotors. It was wound with two layers of DCC copper wire No. 36, like some of the earlier magnetic rotors, and was suspended in the usual way. Its natural frequency was about 6.65 cycles/sec.

(1) With a standard current of 135 milliamperes in the solenoid, and with Y and Z fields very nearly balanced, the resonance amplitude

was measured as a function of X. The results of one of the two sets of concordant observations are given in Fig. 16–1.

(2) With the same solenoid current, and with the X and Z fields very nearly balanced, the amplitude was measured as a function of Y. The results of one of the two concordant sets of observations are given in Fig. 16–2.

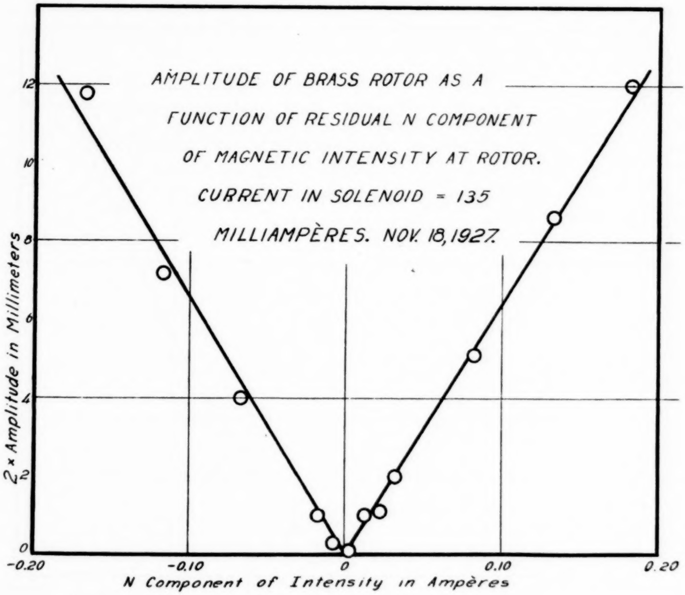

FIG. 16–1.

(3) With X and Y balanced, it was found that Z produced no effect.

(4) With Y and Z balanced and X about ⅓ the northerly component of the earth's intensity, the amplitude was measured as a function of the solenoid current. The results are given in Fig. 16–3.

(5) A current of 10^{-5} ampere (rectangular wave) at the frequency of resonance was sent through the small compensating coils, and produced a deflection amplitude of 6.4 cm. The amplitude of the first harmonic of the torque was then about $1.4 \times 1.4 \times 10^{-3}$ dyne cm.

From this result and Fig. 16–1 it follows that the X component of the earth's horizontal intensity uncompensated would produce on the rotor winding when its current is 135 milliamperes, a torque whose amplitude is about $1.4 \times 2 \times 10^{-3}$ dyne cm.

In the same way, from Fig. 16–2, it follows that the whole uncompensated Y component of the earth's horizontal intensity would produce a torque with amplitude about $1.4 \times 7 \times 10^{-5}$ dyne cm.

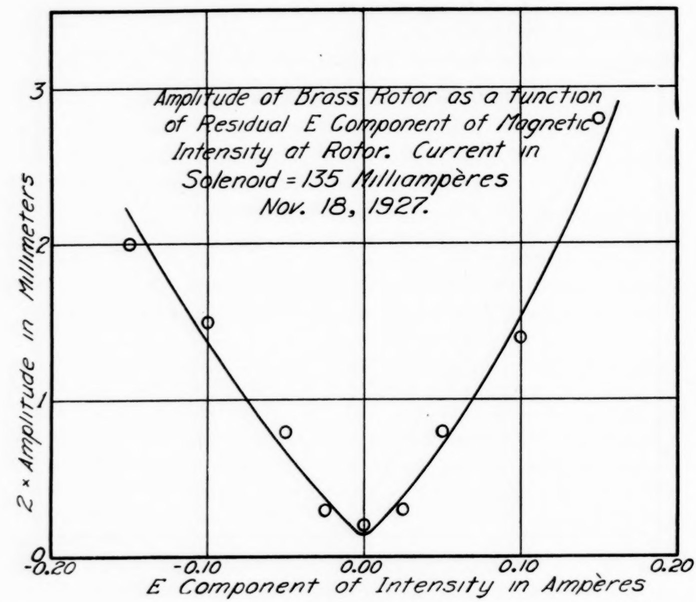

FIG. 16–2.

The amplitude of the first harmonic of the gyromagnetic torque on one of the small permalloy rotors when nearly saturated in the experiments, is about $1.4 \times 1.7 \times 10^{-3}$ dyne cm, and the gyromagnetic torques on most of the other rotors are greater, as they have greater moments.

Thus an error of one per cent in setting X would produce on the brass rotor a torque only about 1 per cent of the torque on the perm-

alloy rotor; and an error of one per cent in setting Y would produce on the brass rotor a torque of only about one twentieth of one per cent of the gyromagnetic torque on the permalloy rotor.

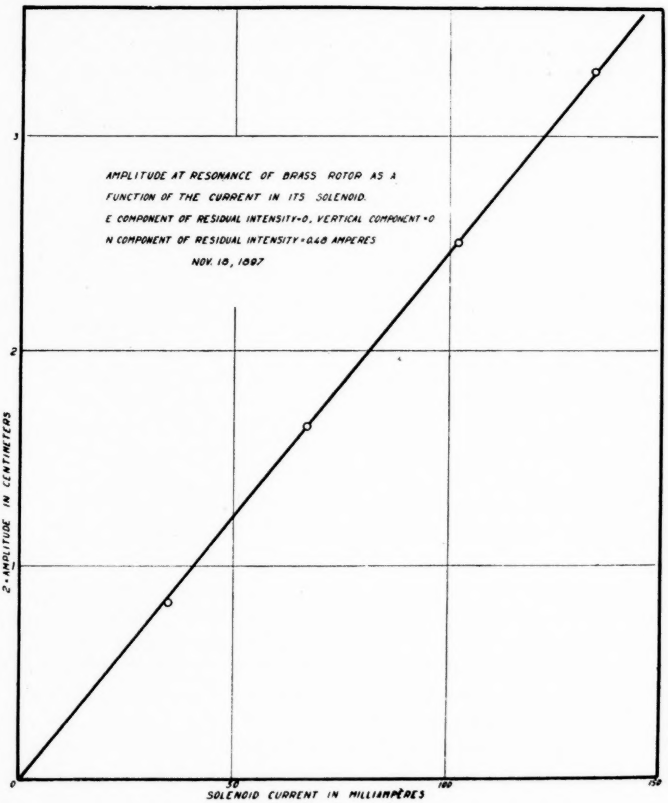

FIG. 16–3.

Similar torques act on the coils of the magnetic rotors; but even when appreciable they have no effect on the determination of ρ, as already explained.

The small residual amplitudes in Figs. 16–1 and 16–2 are doubtless due in part to electron inertia.

In addition to these experiments many additional experiments have been made on this rotor (original and rewound), and a rotor of glass, already referred to. There are minute disturbances which often occur and are not yet wholly explained; nevertheless, the observations have been sufficient to measure, with an error of only a few per cent, the electron-inertia torque of § 15, which is itself very small in comparison with the gyromagnetic torques investigated in this paper. The disturbances referred to cannot therefore affect seriously the value of ρ obtained in this work.

§ 17. F. *Torque due to magnetostriction.* Since the change in rotor length due to magnetization is independent of the direction of the magnetization, no magnetostrictive torque of the same period as that of the first harmonic of the magnetization can arise if the two half-cycles of magnetization are similar. If they are dissimilar, however, either because of a residual vertical magnetic moment, or because the two half-cycles of magnetizing current are dissimilar, there will be an elongation with a first harmonic of the period of the magnetization. For effects of upper harmonics see § 45.

Hence if the rotor is unsymmetrical, or if the twist of the suspensions depends on their tensions, as it is found to do with German silver, there will be a torque of magnetostrictive origin upon the rotor, whose first harmonic may be written

$$h = H \sin (\omega t + \theta) \qquad (17\text{–}1)$$

If when the two half-cycles of current are similar the residual magnetization is reversed (as in most cases by reversing the earth's residual vertical intensity; or, when the half-cycles are dissimilar but the residual moment vanishes, if the connections between battery and commutator are reversed, or the generator field current is reversed, or the connections between the terminals of the rotor coil and the rest of the circuit are reversed, so that the half-cycles producing positive and negative magnetization of the toror are reversed—the torque changes sign without change of magnitude. Its effect can thus be eliminated by taking observations in which such reversals are periodically made.

By driving the generator or commutator at such speed as to produce a current with half the natural frequency of the rotor system, enormous amplitudes due to magneto-striction can sometimes be obtained.

§ 18. *Equation of motion and fundamental formula* when the magnetizing coil is wound on the rotor, and effects D and E of § § 14 and 15 are neglected. If the frequency of the first harmonic of the current in cycles per second is denoted by $\omega/2\pi$ it is clear from § § 10–13 that the equation of motion is

$$(G + C) \cos \omega t - CKA \sin (\omega t + \beta) + \omega \dot{K} A \frac{\omega_0^2}{\omega^2} \cos (\omega t + \beta)$$
$$+ (E_h + E_z) \sin (\omega t - \alpha) + B \sin \omega t$$
$$+ (I + \Lambda) \sin (\omega t + \gamma) + H \sin (\omega t + \theta)$$
$$= \omega KA \cos (\omega t + \beta) \qquad (18\text{–}1)$$

From this equation we obtain for the components of the amplitude of the resultant impressed electromagnetic torque in phase with g and leading g by a quarter period, respectively, the quantities

$$T_{11} = G + C - (E_h + E_z) \sin \alpha + (I + \Lambda) \sin \gamma + H \sin \theta$$
$$= KA \left\{ \frac{\omega^2 - \omega_0^2}{\omega} \cos \beta + \sigma \sin \beta \right\} = UKA = \psi A_{11} \qquad (18\text{–}2)$$

and

$$T_{\perp} = - B - (E_h + E_z) \cos \alpha - (I + \Lambda) \cos \gamma - H \cos \theta$$
$$= KA \left\{ \frac{\omega^2 - \omega_0^2}{\omega} \sin \beta - \sigma \cos \beta \right\} = VKA = \psi A_{\perp} \qquad (18\text{–}3)$$

where U and V are abbreviations for the expressions in curled brackets, A_{11} and $A_{\perp}$ the amplitudes of the deflection which would be produced by the two components separately, and ψ is a constant. The amplitudes A_{11} and $A_{\perp}$ are connected with A by the relation

$$A_{11}^2 + A_{\perp}^2 = A^2 \qquad (18\text{–}4)$$

In the work described here (except special tests) $E_z \sin \alpha$ and $\Lambda \sin \gamma$ are negligible. Cancelling them, we obtain from equation (18–2)

$$G + C + H \sin \theta - E_h \sin \alpha + I \sin \gamma = \psi A_{11} \qquad (18\text{–}5)$$

If G, θ and ω are kept constant, while the torsion-head is turned through the angle π (from azimuth A to azimuth B) and the connections between the torque coil and the rest of the induction circuit are reversed, and if the amplitude of the current in the coil is given the value C', we obtain in place (18–5) the equation

$$G + C' + H \sin\theta + E_h \sin\alpha - I \sin\gamma = \psi A'_{11} \qquad (18\text{–}6)$$

By addition of (16–1) and (16–6) we obtain

$$2G + C + C' + 2H \sin\theta = \psi(A_{11} + A'_{11}) \qquad (18\text{–}7)$$

If now, with G and ω still unaltered, one of the reversals mentioned in § 17, F is made, changing θ by π, we have for current amplitudes C and C' corresponding to azimuths A and B, as above,

$$G + C_1 - H \sin\theta - E_h \sin\alpha + I \sin\gamma = \psi A_{1\,11} \qquad (18\text{–}8)$$

and

$$G + C_1' - H \sin\theta + E_h \sin\alpha - I \sin\gamma = \psi A_1'_{11} \qquad (18\text{–}9)$$

which together give

$$2G + C_1 + C_1' - 2H \sin\theta = \psi(A_{1\,11} + A_1'_{11}) \qquad (18\text{–}10)$$

Combining (18–7) and (18–10), and denoting the mean values of the current and deflection amplitudes by $\bar{C}$ and $\bar{A}_{11}$, we obtain finally

$$G + \bar{C} = \psi \bar{A}_{11} \qquad (18\text{–}11)$$

from which the effects of all disturbing torques (except D and E) are eliminated and which is the fundamental formula sought.

§ 19. *The null-graphical method.* This has already been treated for the ideal case in which all disturbing torques vanish. In a single experiment the amplitude A of the deflection may then be reduced to zero by making c oppose g and giving R a suitable magnitude R_0.

When disturbing torques are present the amplitude $A_\perp$ is in general appreciable, so that only a minimum amplitude ($A_\perp$ itself) can be obtained by varying R. A_{11} is then zero. Four experiments are necessary, two with values of θ differing by π for each of the two azimuths A and B. The effects of all the disturbing torques (except D and E) are then eliminated by using equation (18–11), which becomes simply

$$G = -\bar{C} \quad \text{or} \quad -\rho = \Gamma_0\gamma_0 m_0 \overline{\left(\frac{1}{R_0}\right)} \qquad (19\text{–}1)$$

when $\overline{\left(\frac{1}{R_0}\right)}$ is the mean value of the reciprocals of the four resistances necessary to produce the four minimum amplitudes of deflection.

§ 20. *Large deflection method I.* In the determination of ρ by this method a comparison is made between the amplitude A_G of the

deflection produced by the gyromagnetic torque when there is no current in the torque coil ($C = O$), and the amplitude A_C of the deflection produced by a current $q = Q \cos \omega t$ in the torque coil when there is no current in the magnetizing coil ($G = O$). As in the method just described, four experimental arrangements are necessary to eliminate all the disturbing torques (except D and E); and in each case it is also necessary to measure $A_\perp$ and to calculate A_{11} from A and $A_\perp$ by equation (18–4). When $C = O$, equation (18–11) and (18–4) give from the four arrangements

$$G = \psi \bar{A}_{11} = \psi \sqrt{\overline{A^2 - A_\perp{}^2}} = \psi A_G \tag{20–1}$$

and when $G = O$,

$$C = \psi A_C \tag{20–2}$$

Since $G = -\rho \omega M$, and $C = \Gamma_0\ m_0 Q$, the last two equations give

$$-\rho = \frac{\Gamma_0 m_0 Q}{\omega M} \cdot \frac{A_G}{A_C} \tag{20–3}$$

When rectangular waves are used, Q and M are the amplitudes of the first harmonics, or $4/\pi$ times the steady values to which the current and magnetic moment quickly rise.

The quantities $A_\perp$ are usually so small in comparison with the corresponding quantities A that the latter, which are directly observed, can be used for A_G or A_C without appreciable error. For equal amptitudes the error vanishes completely.

The method is open to the objection that it requires the precise measurement of a number of quantities; nevertheless it is capable of considerable precision. The precision is, of course, greatly enhanced by using resonance. The method is open to the additional objection that the harmonics in q and c, which are not now, as in the mull method (and other deflection methods), produced by mutual induction between the primary and secondary circuits, are not identical with those in g; so that formula (20–3) is from this cause not quite exact. On account of resonance, however, and the great preponderance of the first harmonic in the angular momentum, the error from this source is negligible. No such harmonics are appreciable.

§ 21. *Methods with the magnetizing coil fixed to the earth.* The equation of motion and the other equations applicable to this case are identical with those already developed except (1) that the torque i and its phase angle γ are replaced by the torque d and its phase angle

δ; and (2) that there is added another torque $j = J \sin(\omega t + \zeta)$ (§ 12, B, (b)) which may be treated precisely like the torque h, being eliminated in the same way. The effect of the torque d, which changes sign when the torsion-head is turned through 180°, is eliminated in the same way as the effects of the torques i and e_h. While these latter torques are very minute, however, the torques d and j are often large, so that the apparent values of ρ for the two azimuths and the two positions of the reversing switch are often considerably different.

§ 22. *Corrections due to mutual induction.* In what precedes the variable part of the magnetic flux through the induction circuit has been taken as $\varphi = \mu\gamma_0$, that is the flux through the induction solenoid due solely to the magnetization of the rotor.

The magnetizing coil, however, also produces a flux φ' through the solenoid, and the magnetizing coil and rotor together produce a flux φ'' through the torque coil, which is a part of the induction circuit.

The apparatus was always so symmetrical that the flux φ'' either vanished or was entirely negligible in comparison with φ, as careful tests showed.

In the fixed solenoid method, and sometimes in the moving solenoid method, the flux φ' was compensated by means of a pair of coils, one similar to the induction solenoid, but longer, the other within the first, having nearly the same magnetic moment per unit current as the magnetizing coil. When φ' was appreciable, compensation could always be effected with much greater precision than necessary by moving one of the coils relatively to the other, or adding or subtracting a few extra turns, until the mutual induction vanished.

This was quite important in the fixed solenoid method as there φ' and φ were always of the same order, of magnitude, though φ' was always less than φ.

In the case of the moving solenoid, however φ'/φ was always small, ranging from about 0.2% to about 1%.

Since with the rectangular wave commutator the current was practically in phase with the vertical magnetic moment, the existance of φ', when small, and uncompensated by mutual induction, could always be allowed for by applying to the value of ρ determined on the assumption that $\varphi'/\varphi = o$ a correction

$$\Delta\rho = \rho \times \frac{\varphi'}{\varphi} = \rho \frac{M}{M'}.$$

The connection is clearly always positive. On account of the small-

ness of the correction it could be, and has been, applied in the same way without appreciable error in the case of experiments made with sine waves.

There is also a harmonic flux φ''' though the induction circuit due to the motion of the permanent magnet, with moment m, with respect to the compensating coil. The quantity φ'''/φ however, is entirely negligible. For the iron rotor $I^2{}_{2\cdot4-2}$ its magnitude was about 5×10^{-5}.

When the rotor does not hang symmetrically through the induction solenoid and the torque coil, some of the flux produced by the rotor threads the torque coil. Even with asymmetry far greater than ever existed in the experiments this flux was entirely negligible. Thus, for example, with the center of the torque coil only 10 cm below the end of the rotor $I^2{}_{2\cdot4-2}$ (less than ½ the distance in nearly all the work) and axially displaced 4 mm., the flux through the torque coil was not measurable when the flux through the induction solenoid produced a deflection of about 83 cm.

§ 23. *Experimental curves in the null method.* In making a series of observations by this method, G (and the remainder of T_{11}) is kept constant, while C, otherwise constant, is varied by changing R, which occurs in its denominator. The amplitude of the impressed torque, when no disturbing torques in quadrature are present, is thus $T_{11} - \psi/R$, where ψ is a constant. The deflection amplitude A is proportional to the impressed torque, so that we may write

$$A = \alpha(T_{11} - C) = \alpha\left(T_{11} - \frac{\psi}{R}\right) = \beta - \frac{\gamma}{R} \qquad (23\text{-}1)$$

where α, β, and γ are constants.

The equation (23-1) between A and R gives the two rectangular hyperbolae drawn in Fig. (23-1). The experimental parts of the curves are drawn in heavy continuous and chain dotted lines.

The curves obtained by experiment agree with such hyperbolae closely, except that, when there are disturbing torques in quadrature with the gyromagnetic torque, the curves are rounded near the junction, as indicated in the figure. The abcissa R_0 of the junction at which the minimum amplitude occurs is not affected, only the ordinate. When there are disturbing torques in phase with or in opposition to the gyromagnetic torque, the junction is shifted to the left or right.

§ 24. *Transformation from hyperbolae to straight lines.* If, instead

of plotting the relation between the vibration amplitude and the resistance R, we plot that between the amplitude and the conductance $X = 1/R$, we obtain the straight lines of Fig. (24–1), which are so drawn as to make clear the relation of their parts to the corresponding parts of the hyperbolae of Fig. (23–1).

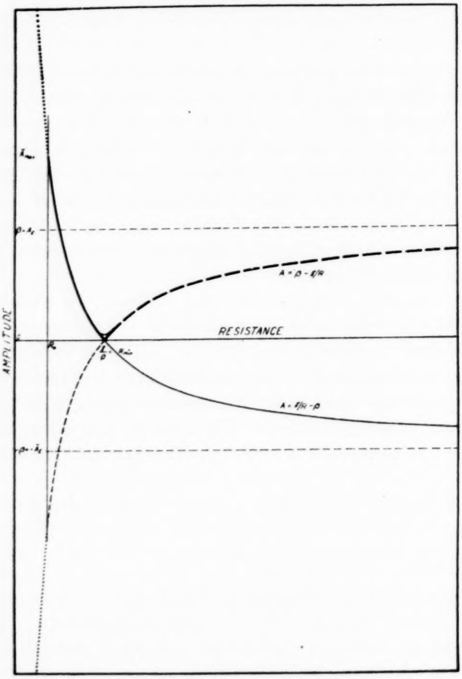

Fig. 23–1.

Each of the straight lines cuts or touches the axis of conductance at the point $X_0 = I/R_0 = \beta/\gamma$. Reproductions of some of the straight lines obtained from actual experiments are given in Fig. (46–1). From these the hyperbolae were drawn, and are given in the same figure. The observed points are given by the small circles.

In finding X_0 graphically by means of such straight lines, it is better

to plot the amplitudes obtained for conductances greater than X_0 as negative quantities, and draw a single straight line, as in the curves of Fig. (46–1).

§ 25. *Method of interpolation from small deflections.* While in much of the earlier work experimental curves, either hyperbolae or straight

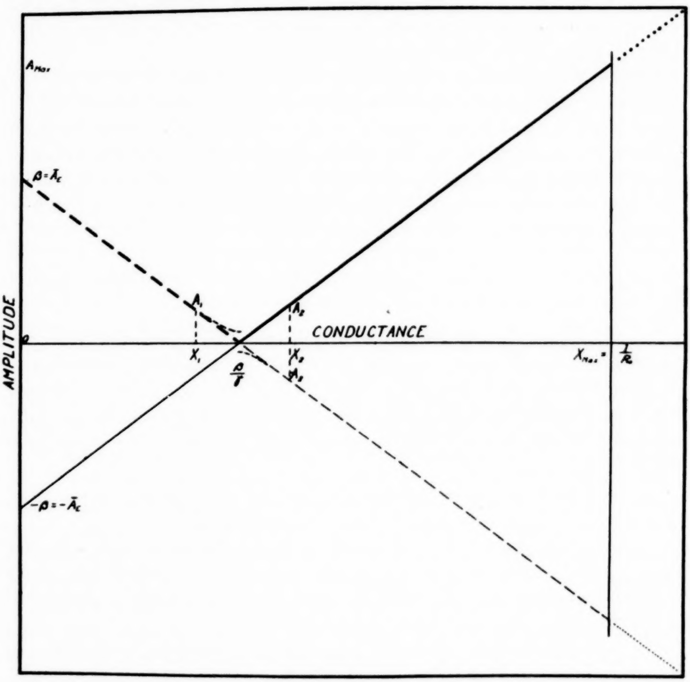

FIG. 24–1.

lines, were drawn for a number of points, particularly in the neighborhood of the minimum amplitude, and the resistance or conductance for the minimum taken from the curves, a different procedure was adopted in some of the work, interpolation being made from a few small amplitudes taken for conductances on opposite sides of that corresponding to the minimum amplitude.

Thus if A_1 and A_2, Fig. (24–1), denote the vibration amplitudes corresponding to conductances X_1 and X_2, less and greater, respectively, than X_0, the conductance for vanishing amplitude, we obtain

$$X_0 = X_1 + (X_2 - X_1)\cdot A_1/(A_1 + A_2) \tag{25–1}$$

When X_1 and X_2 differ by about the same amount from X_0, the same relation holds even when the lines are curved, as in the dotted part of Fig. (24–1), since symmetry still exists.

§ 26. *Large deflection method II.* The same figure leads to a large deflection method far superior to that of § 20 and used in most of the later work. Thus, if we make $X = X_1 = 0(R_1 = \infty)$, we obtain the amplitude A produced by the gyromagnetic torque and the (usually small) disturbing torques; and if we make $R = 1/X = 1/X_2$ about one-half the value for the minimum amplitude A_0, we obtain a nearly equal amplitude A_2. We then have simply

$$X_0 = X_2 \frac{A}{A + A_2} \tag{26–1}$$

As the torques in quadrature with g produce very little effect on the amplitudes when large, results obtained in this way are almost independent of such torques. The effect of the quadrature torques when not negligible and yet not too great can be eliminated by substituting in the formula for the amplitudes A and A_2 the quantities $\sqrt{A^2 - A_0^2}$ and $\sqrt{A_2^2 - A_0^2}$.

As will be seen below, results obtained by this method agree very closely with those obtained by the other methods.

Although formula (26–1) is based on the assumption that only the first harmonic is present in the angular momentum, it holds also when higher harmonics are appreciable, since these harmonics enter in exactly the same way into both A and A_2.

In all methods it is, of course, necessary, in general, to find the X_0 from each of the four different experimental arrangements mentioned above, and to calculate ρ from the mean.

§ 27. *Method of remote induction solenoid.* If we have two rotors A and B exactly alike and wound with magnetizing coils exactly alike, so that equal currents produce equal moments, A may be mounted in the usual way to obtain the gyromagnetic torque, while B, inside the induction solenoid (or a similar solenoid) and at rest, may be mounted at a distance. With the rectangular wave commutator, however, it is unnecessary that the two rotors, or the rotor A and the

rod B, have the same moments or be of the same material, or be wound alike. It is only necessary to know the ratio M_B/M_A of the maximum moments when the two magnetizing coils, in series, are traversed by the same maximum current. In this case we have, in place of (19–1), the equation

$$-\rho = \frac{M_B}{M_A} \Gamma_0 \gamma_0 m_0 \overline{\left(\frac{1}{R}\right)} \tag{27–1}$$

A number of measurements made in this way gave results agreeing closely with those made in the usual manner, which avoids any error in determining M_B/M_A.

§ 28. *The apparatus adjacent to the rotor.* The way in which the principal apparatus near the rotor was mounted is illustrated in Fig. 28–1. The induction solenoid A was held in a special wooden clamp B between the boards CC attached (with ebonite insulation in between) to a heavy wooden frame E resting on a heavy wooden table F. A third board D carried the compensating coil, also insulated with ebonite. The frame and table also supported the coils for neutralizing the horizontal intensity of the earth's magnetic field. It was provided at the top with adjustable diagonal brass rods for correcting distortion; and two boxes HH, for holding sand, were attached to the bottom of the table. The table, together with all it supported, was suspended from the ceiling by twenty-four heavy brass springs attached in equal groups of six each to the wooden beams GG, below the table top, and two similar beams at the ceiling. The lower ends of some of the springs are shown disconnected in the figure. Spirit levels, J and K, in brass cases, and sensitive to 10″ or thereabouts, made it possible to set the apparatus with precision, where once the induction tube, neutralizing coils, etc. had been placed in correct relative adjustment by means of plumb lines, etc. The table was leveled, and stability secured, by means of sand in the boxes HH. The legs of the table nearly touched the floor, and damping of its motion was produced when desired by placing small amounts of cotton waste between the floor and the legs.[12]

The large rectangular frame carrying the coil for neutralizing the vertical intensity of the earth's magnetic field surrounded the other apparatus and was supported from wooden beams at the ceiling by brass wires attached at the bottom to adjusting slides of brass provided with slow motion by screws.

[12] For later improvements in the method of damping see Footnote 10, § 9.

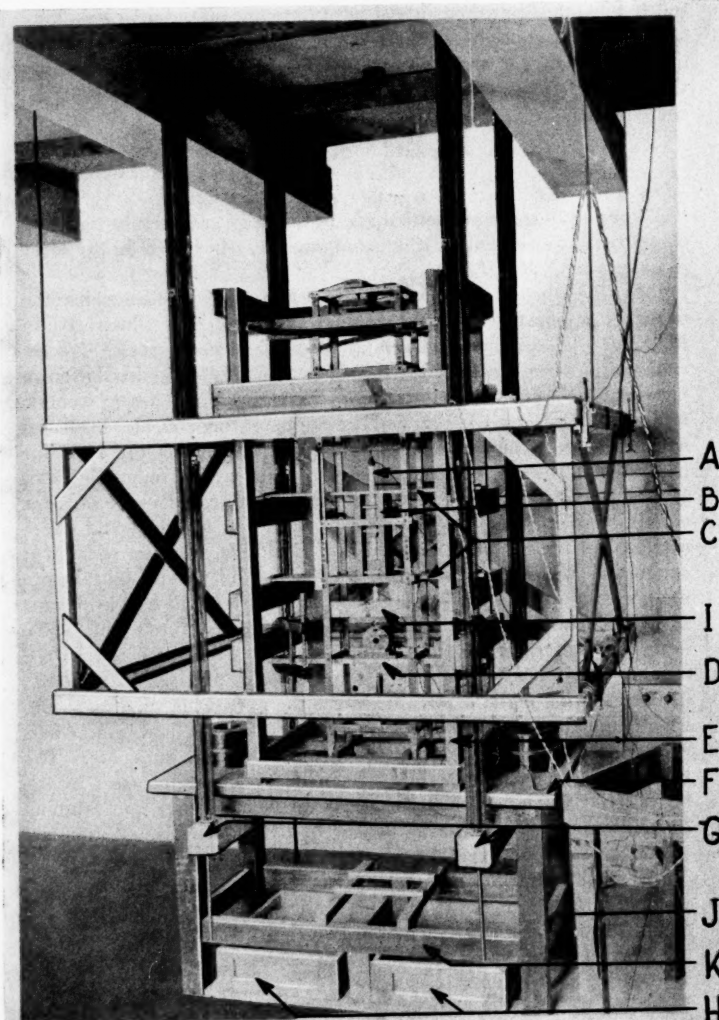

FIG. 28–1. The central apparatus.

At I is a light wooden support carrying a spectacle lens by which, and the mirror on the vibrating apparatus, an image of the vertical filament of an electric lamp is formed on a translucent scale between four and five meters distant.

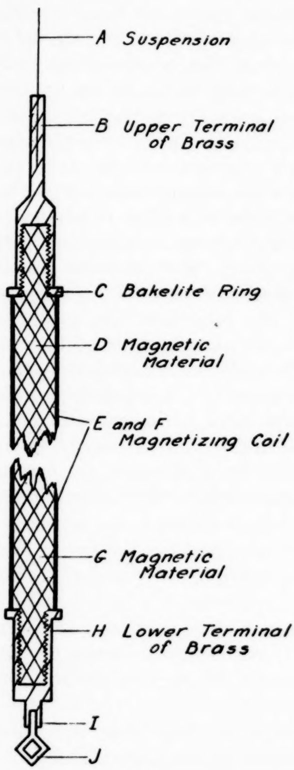

FIG. 29-1. Construction of standard wound rotor.

§ 29. *The rotors.* The magnetic rotors, of annealed Armco electrode iron and Bell Telephone permalloy, were of two types, those with a magnetizing coil wound upon them, and those without a winding.

Most of the rotors with the coil wound upon them were constructed as indicated in Fig. 29–1. The whole length of the rotor was about 31 cm, and that of the magnetic material about 27 or 28 cm. An approximately circular and cylindrical rod of the magnetic material was made as nearly straight as practicable. The ends were threaded, and on them were screwed upper and lower terminals of brass of the forms indicated. The upper terminal was drilled and drawn for the suspension of German silver wire, which was very carefully soldered in. If the hole did not fit the wire closely, instead of resorting to drawing, the terminal was sometimes symmetrically squeezed onto the wire with a three-jawed chuck. The lower terminal ended in a rectangular ring, with rectangular cross-section, in which the magnet-mirror holder hung. Between the upper terminal and the main cylinder of the rotor was a small bakelite washer which was screwed on to the rotor and pressed tightly against its shoulder by the upper terminal as it was screwed into place. The rotor was closely wound, between the bakelite ring and the lower terminal, with from two to eight layers of silk covered wire. No. 36 was at first used, No. 40 later; and in almost all cases the terminals were twisted together, drawn through a small hole in one side of the washer, and wound in a spiral about the upper terminal and suspension, being finally soldered to two small brass rods in the torsion head, as indicated in Fig. 6–1. Everything was done to make the whole body as nearly symmetrical about the axis as practicable. As the best rotors are exceedingly soft, it was usually necessary to restraighten them after winding.

In some cases the bakelite washer was replaced by a ring of brass, which was then made integral with the upper terminal. This is, of course, a better construction mechanically, but endangers the insulation.

In one rotor the upper and lower ends of the magnetic material, above E and below F in the figure, were replaced by brass, so that the whole magnetic part of the rotor was in an approximately uniform field.

One rotor was similar to those first described except that the magnetic part was about 8 cm shorter, and the lower terminal, of brass, about 8 cm longer. This made it possible to mount the rotor in the fixed magnetizing solenoid in such a way that the magnetic material was wholly inside the uniform part of its field.

The rotors without windings were similar to the last, but slightly shorter; they could, of course, be used only with the fixed-coil methods.

Other rotors had their upper terminals shaped for suspension from a flat strip instead of a round wire. One rotor was similar to the first except that the magnetic material was wholly replaced by brass. Another was of glass, with brass caps at the ends.

The rotors without windings were about 1.5 mm. in diameter. The diameters of the cores of the magnetic rotors with windings ranged from about 2 mm. to 3.2 mm. The core of the brass rotor was 2.4 mm. in diameter, that of the glass rotor 4.8 mm.

In the designations of the rotors the symbol I refers to iron, and P to permalloy; the subscript s prefixed indicates that the rotor was of the shorter type, all without this subscript being of the longer type; the subscript 36 prefixed, that it was wound with No. 36 wire, all others being wound with No. 40 wire, or being unwound; the exponents 1 and 2, that the rotor core was used in two states, before and after rewinding. The first subscript suffixed indicates the diameter in mm., the second the number of layers of wire. Thus, for example, ${}_{36}I^{1}_{2.4-2}$, means an iron rotor 2.4 mm. in diameter of the standard length wound with 2 layers of No. 36 wire, and indicates that the rotor was later rewound.

Photographs of two of the rotors, with their suspensions and torsion heads, are reproduced in Figs. 29–2 and 29–3. Resonance curves of two rotors, one wound, the other unwound, are given in Fig. 29–4.

§ 30. *The magnets, mirrors, and magnet-mirror holders.* The small magnets and mirrors indicated in Fig. 6–1 were carried on long brass holders as shown in Fig. 30–1. Two holders Nos. 1 and 2 were used in the earlier work. They were 23 cm long and otherwise nearly alike except that the straight part of one was of rod about 2.4 mm. in diameter, while the straight part of the other was about 1.5 mm. in diameter. At the upper end is a rectangular hook, at the lower a rectangular ring or hook, each with rectangular cross-section. The upper hook fits into the ring in the lower end of the rotor, the lower ring or hook into the upper hook of the lower suspension. At the distance of 8 cm from the top of one holder and 11.5 cm from the top of the other, two small thin brass plates were soldered parallel on opposite sides of the rod. By means of small projections and a small amount of universal wax, the plates carried two small parallel silver-on-glass mirrors. The area of each mirror was about 6×3 mm.[2] For a centimeter or so, at the mean distance of 20 cm from the top, the rod was flattened and two similar groups of small hardened steel rods normal to the brass rod, and nearly parallel to the mirrors,

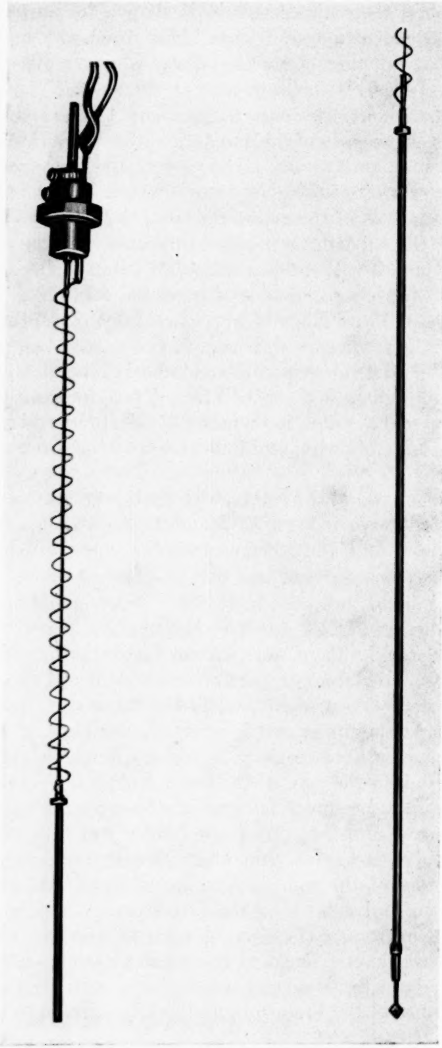

FIG. 29–2. The rotor $I^2_{2.4-2}$ with suspension and torsion head.

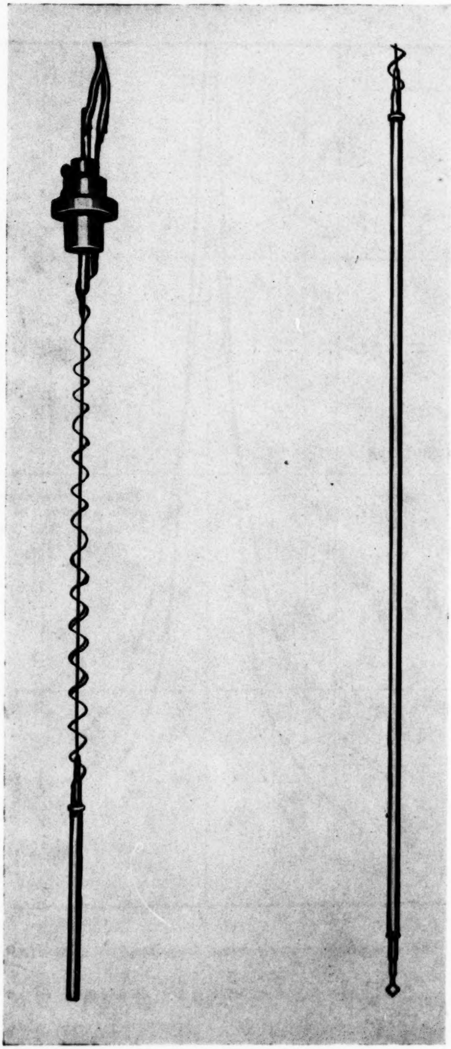

Fig. 29–3. The rotor $P^2_{3\cdot2-4}$ with suspension and torsion head.

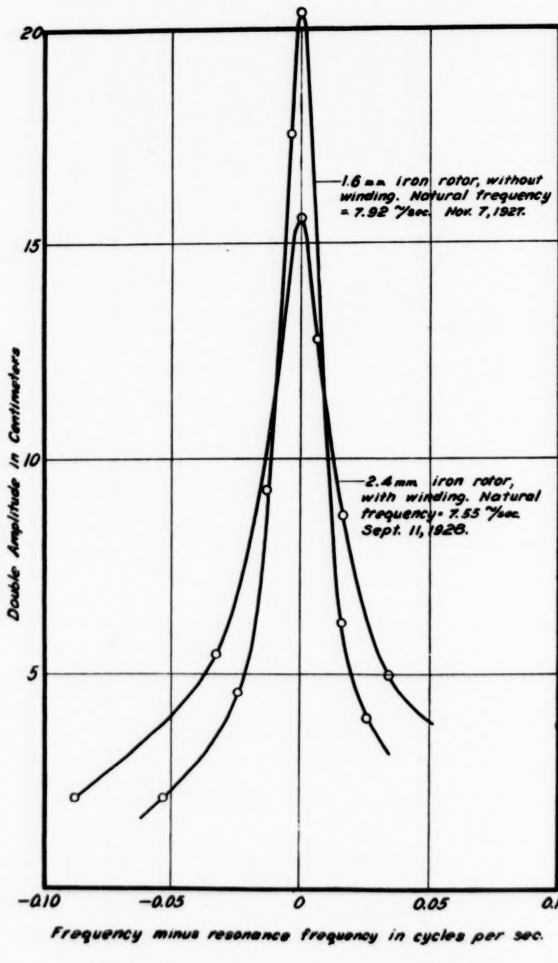

FIG. 29–4. Resonance curves for two types of rotor.

Fig. 30–1. Magnet-mirror holder No. 2.

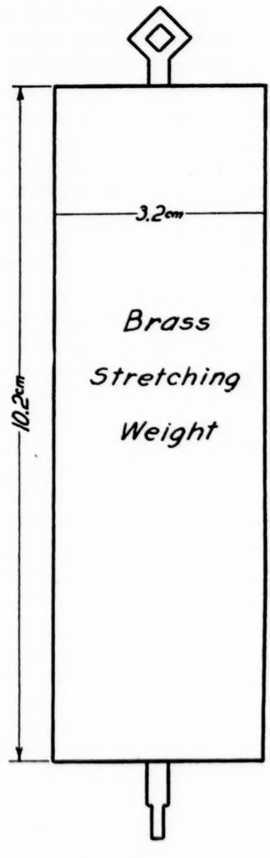

Fig. 31–1.

were securely cemented on with shellac. The steel rods on the thicker holder were similar to those used in the magnetometer during the latest work on magnetization by rotation. They were 5 mm. long, 0.17 mm. thick and 0.35 mm. wide. They were mounted with their axes 1 mm. apart in two groups, one on each side of the rod. In

all the later work there were 6 magnets in each group, though as many as 8 were sometimes used earlier. The magnets of the completed group were magnetized together in the intense field of a suitable electromagnet. In the case of one of the holders the angle between the planes of the magnets and the planes of the mirrors was negligible; in the case of the other, this angle was about 4 degrees. The mirrors gave very good images with lamp and scale at distances greater than 4 m.

A third holder, used in all the latest work, was similar to the second, except that it extended 10 cm farther below the magnets and that the mirrors were thicker, had greater areas (nearly 6×7 mm.) and were mounted strictly parallel to the magnets, each resting against and being cemented to three points in its brass support. The surfaces were at first silvered, later gilded.[13]

§ 31. *The upper suspensions, lower suspensions and stretching weights.* All the upper suspensions were of German silver wire No. 28, 30, 32, or 34, or of strip rolled from No. 28 German silver wire. Most of them were about 15 cm. long. Several lower suspensions of German silver wire and strip, were used. They were all of about the same length, viz. 7.5 cm, but had different torsional constants in order to vary the frequency of the vibrating system conveniently. The wire of each suspension was soldered to rectangular hooks of the sort already described, the upper hook fitting into the ring or hook at the bottom of the magnet-mirror holder, and the lower into a rectangular ring at the top of the stretching weight.

Two weights, one of copper, the other of brass, were used. Each had a central rectangular ring at the top and a small axial rod of German silver at the bottom. The masses of the copper and brass weights were 361 and 685 grams, respectively. The latter is illustrated in Fig. 31-1. Weights were interchanged to move spurious resonance peaks farther from that at the vibrator's fundamental frequency.

§ 32. *Induction solenoids.* Several induction solenoids were used in the course of the work. The solenoid used in a great deal of the earlier work with magnetizing coil attached to the rotor was wound on a circular cylindrical bakelite tube 56 cm in length and with internal and external diameters of ½ and ¾ inch, respectively. The coil, of No. 30 cotton enamel copper wire, was wound with precision on the lathe in a single layer with a pitch of 44 turns per inch, successive turns not touching one another, and was shellaced

[13] For the gilding, which was beautifully done, I am indebted to the kindness of Dr. C. Hawley Cartwright.

in place. The coil was divided into two parts, one (the upper) 48 cm long, the other (the lower) 8 cm long; and the bakelite tube was likewise divided into two parts, one screwing into the other and bringing the two coils very near together. The two parts of the coil were provided with thin flexible twisted leads, and all were properly bound, first with friction tape, then with linen tape.

The torsion-head carrying the rotor fitted exactly in the top of the tube, and the ring at the lower end of the rotor then protruded a few millimeters from the lower end of the upper and longer part of the tube. After the rotor was in position, the magnet-mirror holder was slipped through the shorter tube and hung on the rotor's ring; then the shorter tube was screwed into the longer, and the leads properly connected together through a reversing switch.

Another tube, designed for work with both moving coil and fixed coil methods, and used in nearly all the later work, was similar to that just described except that its internal and external diameters were 1 inch and 1¼ inch, respectively, and that the short part slipped over the lower end of the longer part, instead of being screwed into it. The tube carrying the magnetizing coil generally used in the fixed coil method was held coaxially in this tube by means of amber bushings; and the torsion head carrying the rotor was then mounted in the upper end of this latter tube. It could also be mounted in a special amber bushing mounted directly in the top of the induction solenoid. In this case the fixed magnetizing coil was removed. In either case the ring at the lower end of the rotor was a few mm. below the lower end of the upper part of the induction solenoid, just as in the case described above. The arrangement is illustrated in Fig. 32–1. The inductance of this solenoid was about 2.4×10^{-3} henry.

Two of these coils were measured after long use, and each pitch found to be uniform to a degree much beyond the requirements of this work, and its mean value in exact agreement with that expected from the lathe, screw, and gears.

In the earlier part of the work other solenoids, less precisely wound, with approximately 70 turns per inch, 46 turns per inch, and 136 turns per inch, were used.

§ 33. *Magnetizing coils.* In the early part of the work, after preliminary experiments, the coils wound on the rotors consisted of two layers of No. 36 (SSC) copper wire, closely wound. The rotors wound later had either two, four or eight layers of closely wound SSC No. 40 copper wire. We were unsuccessful in getting proper insulation with enamelled wire.

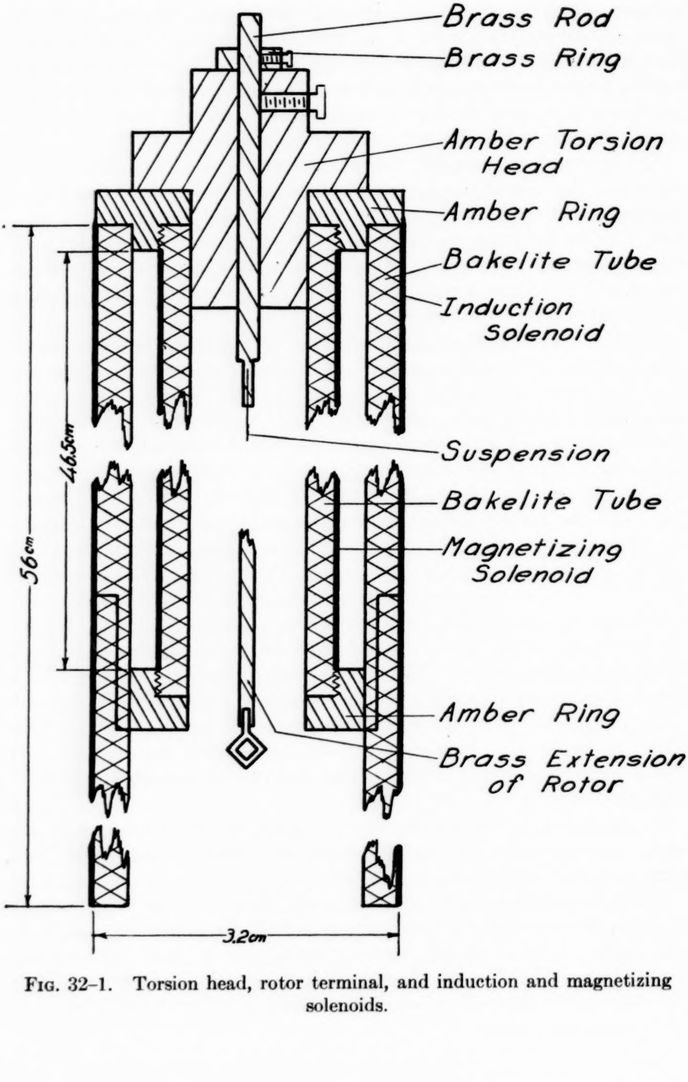

FIG. 32-1. Torsion head, rotor terminal, and induction and magnetizing solenoids.

The magnetizing coil used in nearly all of the work done by the fixed coil method was wound of No. 28 DDC copper wire on a circular cylindrical bakelite tube with amber bushings at the ends. It was 46.5 cm long (between amber bushings). The coil was in two layers, each wound to pitch very nearly 46 turns per inch. The inner and outer diameters of the tube were $\frac{1}{2}$ inch and $\frac{3}{4}$ inch, respectively. The coil was provided with flexible leads, and all was properly taped.

For the production of stronger fields in the fixed coil method, another magnetizing solenoid thirty-eight cm long and with eight layers of the same kind of wire was wound on a thin glass tube of small diameter provided at the ends with amber bushings, by which it was held in place in the larger induction coil symmetrically about the rotor. The inner and outer diameters of this coil were 7 mm. and 14 mm., respectively.

§ 34. *The torque coil.* The small torque coil in the induction circuit acting on the suspended magnet was wound in two halves on bakelite bobbins of the dimensions indicated in Fig. 34–1. Each half coil was approximately 1.7 cm wide, and had internal and external diameters of 3 cm and 5 cm, respectively. Each coil contained 20×28 turns of No. 28 DCC copper wire. The bobbins, *A*, *B*, were held by two circular bakelite discs *C*, *D*, 10 cm in diameter and wound with subsidiary coils, the use of which, however, was abandoned early in the course of the work. The larger discs were carried by a bakelite base *E*, by which they were attached either to the framework carrying the vibrating apparatus, or to a special magnetometer for determining the constants of the coils. The resistance of the compensating-coil was about 33 ohms and its inductance about 0.004 henry.

§ 35. *The compensating coils* used to balance the induction effect of the magnetizing coil on the induction circuit were in the form of long solenoids. The secondary coil, 83 cm long, was wound with precision to the pitch 44 turns per inch of No. 30 cotton enamel wire on a circular cylindrical bakelite tube with internal and external diameters of 1 inch and $1\frac{1}{4}$ inch, respectively. As it turned out to be necessary, a few extra turns were wound around the secondary solenoid and connected in series with it.

The primary coils were duplicates of the 46.5 cm and 38 cm magnetizing coils described above, and similarly mounted, except that in the case of the first the amber bushings at both ends were of the proper diameters to hold the coil coaxially anywhere within the secondary.

§ 36. *Equipment for the production of the magnetizing current.* The current was obtained at will either from a 2 KW sine wave alternating current generator or from a storage battery and one of several commutators giving approximately rectangular waves. The

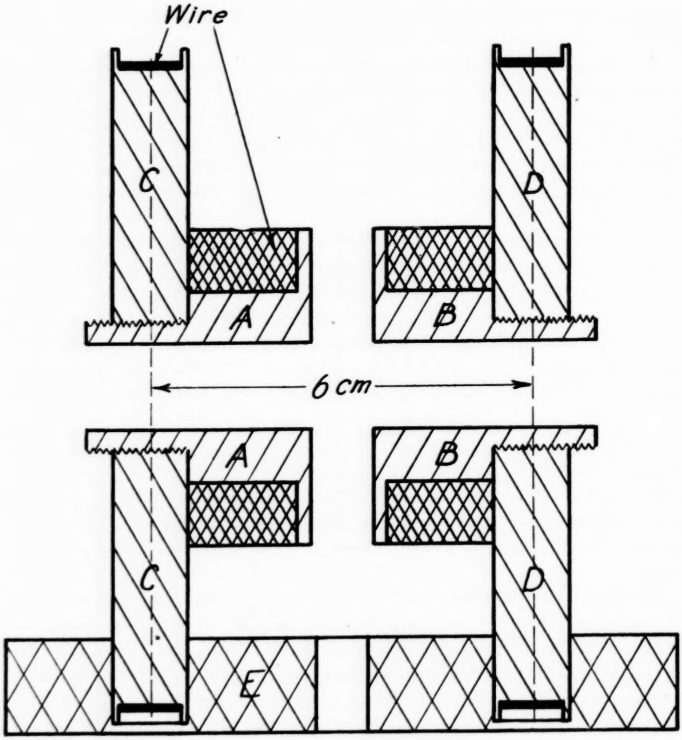

Fig. 34–1. The torque coils.

rotating part of the commutator was mounted on the shaft of the generator; and the brush-holder, of bakelite, wasr igidly secured to its frame.

The generator was driven, through two spiral gear-boxes in series,

by a 3 H.P. shunt-wound motor supplied from a storage battery of large capacity. The first gear-box reduced the speed of its driving shaft to 1, 1/2, 2/7 that of the motor; the second box reduced the speed in the same ratios. Thus the shaft of the generator, for any speed of the motor, could be driven at various speeds from that of the motor to 4/49 that of the motor, whose normal speed was about 30 revolutions per second.

The gear-boxes are the same as were used in the last investigation on magnetization by rotation previously referred to.

Several commutators making two reversals per revolution were used in the course of the work. The method of construction finally adopted is illustrated in Fig. 36-1. The rotating parts were made

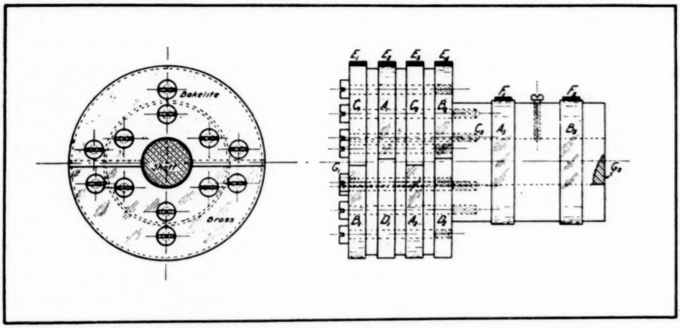

FIG. 36-1. One of the Commutators.

of bakelite and brass. The parts marked C and D were of bakelite; those marked A and B, of brass. All the conductors A_1 were connected together, likewise all the conductors A_2. The two brushes E_1 and E_2 were connected together, and similarly the brushes E_3 and E_4. The cylinders on which the brushes pressed were about 4 inches in diameter. In one commutator the insulating gaps were about 1/8 inch across; in two others about 1/64″ and 3/4″ respectively.

A fourth commutator was constructed in much the same way except that each circle contained three bakelite and three brass sectors, so as to produce 6 reversals per revolution. The insulating gaps of this commutator were about 1/32 inch across.

The brushes first used were of rolled German silver strip so bent and filed at the ends as to make contact over a very narrow strip

of the cylindrical surface. Although these brushes worked well for a time when properly lubricated with vaseline and when the proper pressure was applied, they later gave a great deal of trouble. Laminated brushes of phosphor bronze, bound together with rubber bands or tubing to assist the laminations in the prevention of vibration, and applied tangentially, and also heavy graphite brushes with blunt wedge-shaped ends, pressed against the commutator surface by spiral springs are far superior. Such graphite brushes were used in most of the later work. As the figure shows, all the brushes were applied along the same element—and an upper element—of the cylindrical commutator surface.

All the apparatus described in this section was mounted in the sub-basement of the laboratory, two stories below the room in which the vibration experiments were made.

§ 37. *Frequency control.* Inasmuch as all the methods of experimentation used were resonance methods, it was necessary that the current frequencies, and therefore the motor speeds, should be maintained very nearly constant for long intervals. For this purpose, the method of motor control devised by F. Wenner some years ago, but never published by him[14] so far as I know, was adopted, with comparatively slight modifications, and has proven very satisfactory.

In the simplest form of Wenner's method, a resistance coil in series with the motor field coils is automatically short-circuited one quarter of the time during normal operation when the motor shaft makes exactly one revolution in the period of an electrically driven tuning-fork. This tuning-fork and a commutator on the motor shaft together increase the time of short-circuit (thus strengthening the mean motor field) if the motor gains on the fork, and decrease the time of short-circuit (thus weakening the mean motor field) if the motor lags behind the fork. The motor and the commutator are thus automatically made to rotate in the period of the fork with fluctuations of the phase difference between fork and commutator which may be made very slight.

A diagram of the arrangement used in this work is given in Fig. 37–1, where d is the resistance coil automatically short-circuited, α the tuning fork and (2)–(5) the commutator.

The tuning fork contact $\alpha\beta$ is closed one-half the time, as indicated by the imaginary equivalent brush and rotating circle (1). One

[14] A brief description is given in F. A. Laws' *Electrical Measurements*, p. 432 (N. Y., 1917).

semi-circle (black) is supposed to be of metal and permanently connected with the fork α, the other (white) to be of insulating material, while the wire β is connected to the brush. The whole circle rotates uniformly with the period of the fork in the direction of the arrow.

In the type of commutator (2)–(5) usually used, there are four coaxial circles of brass and bakelite firmly screwed together and mounted on the motor shaft, but all insulated therefrom. Circle (2) is of brass, while the remaining circles, or rings, are half brass and half bakelite, the bakelite halves being shown white. The metal parts of all four circles are in permanent electrical connection. The four circles are pressed upon by four laminated phosphor-bronze

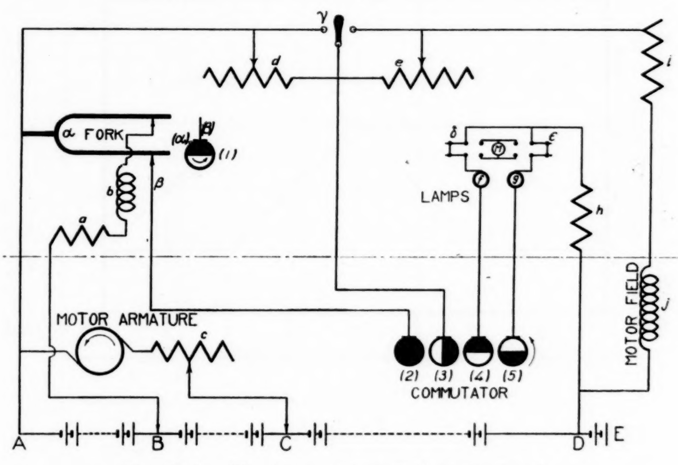

FIG. 37–1. The frequency control arrangement.

brushes along a line parallel to the shaft. The phase relations between the conducting semi-circles are permanently as indicated in the figure, (4) being a quarter period ahead of (3), and (5) being a quarter period behind (3). In the figure, (3), the semi-circle which short-circuits the resistance coil, is a quarter period behind the tuning fork (1)—the normal phase difference for steady running. In this position, as the figure shows, the resistance d is short circuited just one quarter of the time when the switch γ is open or thrown to the right.

In order to use the same tuning fork in controlling the motor when driven at very low speeds, a second type of commutator was built on the same principles as the first, but with the semi-circles (3), (4), (5) replaced by quadrants in the same phase relations. With this commutator the fork of course controlled the motor when driven at half its own frequency.

In Wenner's apparatus two similar incandescent lamps f and g connected to the brushes of circles (4) and (5) and, through switches δ and ε and (if desired) a rheostat h, to the battery serve to indicate the phase relation between (3) and (1).

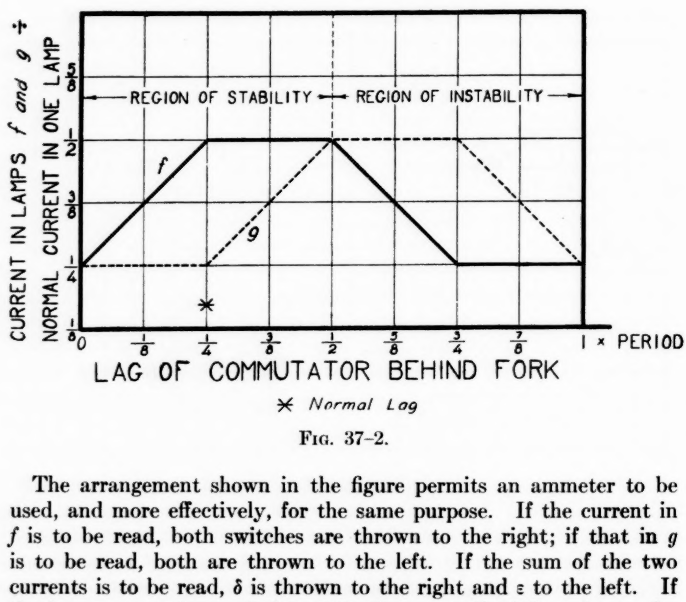

FIG. 37-2.

The arrangement shown in the figure permits an ammeter to be used, and more effectively, for the same purpose. If the current in f is to be read, both switches are thrown to the right; if that in g is to be read, both are thrown to the left. If the sum of the two currents is to be read, δ is thrown to the right and ε to the left. If the lamps are to be used alone, δ is thrown to the left and ε to the right. The relations between the currents in f and g and the lag of the commutator (semi-circle (3)) behind the tuning fork (1) are easily worked out from the diagram and are given in Fig. 37-2. In the region of stable running f is greater than g, the difference being a maximum for the normal lag.

When the switches are thrown so that the ammeter reads the sum of the currents in f and g, the relation between current and lag is that given in Fig. 37–3. With this arrangement the current for normal running is $\frac{3}{4}$ the normal current in one branch. If the current is greater, the motor is too slow; if it is less, the motor is fast.

To get the motor into synchronism with the tuning-fork, the switch δ is first thrown to the left, thus cutting out the resistance d and inserting in the motor field a resistance $e = \frac{3}{4}\,d$. The effect of e on the speed is the same as that of d during normal running, since then d is short circuited one quarter of the time.

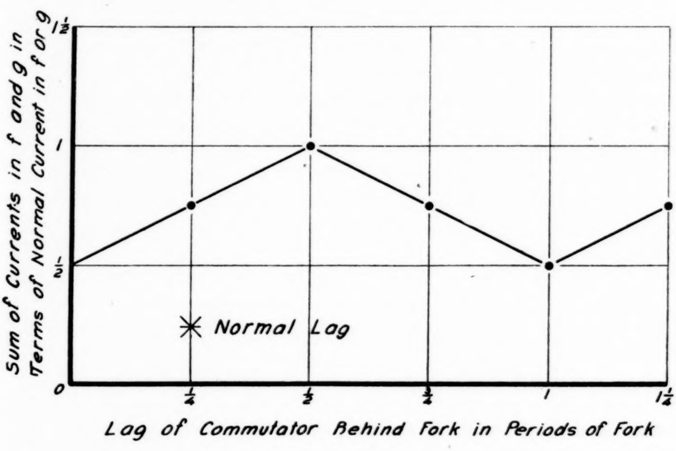

FIG. 37–3.

The motor is made to run at very nearly synchronous speed by adjusting the armature and field voltages AC and AD, and the armature and field resistances c and i. As synchronism is nearly approached the fluctuations of the lamps or ammeter become very slow. When this state has been attained, and when the current in f is the maximum and that in g the minimum (or their sum equal to the mean of the extreme values of the current when the motor is out of synchronism), the switch γ is thrown to the right. Automatic control then sets in. If the motor has been in operation at nearly the correct speed for some time, so that steady temperatures have been reached, the

synchronous speed may be maintained for hours, with only slight fluctuations in the phase difference between commutator and fork. With the arrangement corresponding to Fig. 37–3 the fluctuation of phase difference is clearly equal to the fluctuation of the current divided by twice the difference between the minimum and maximum currents when the motor is thrown slightly out of synchronism.

The tuning-fork was one of the Western Electric Company's standard electrically driven forks having a normal frequency of about 48 periods per second. It was provided with several pairs of brass and bronze weights which, one or more pairs at a time, were symmetrically clamped by steel screws to the prongs in suitable positions to give (through the gear-boxes) frequencies equal to the normal frequencies of the vibrating systems (and other frequencies for certain tests). Rough adjustments could be made by changing or moving the heavier weights near the free ends of the prongs; finer adjustments by moving smaller weights at greater distances from these ends. In Wenner's apparatus provision was made for frequency adjustment by continuous alteration of effective prong-lengths. A modified form of Wenner's device was used in the earliest part of this work, but was soon abandoned in favor of the devices just mentioned. The contact $\alpha\beta$ (Fig. 37–1) was so adjusted as to be closed one-half the time when the fork was in operation. The fork was ordinarily operated by a 30 volt battery through 500 ohms external resistance, the mean current through the electromagnet a b being then about 0.02 ampere.

§ 38. *The constants of the induction solenoids.* The mean constant of either solenoid for the full length of any one of the rotors used is very nearly equal to $4\pi \times$ the number of turns per cm, i. e. $4\pi \times 44/2.54 = 217.68$ e.m.u. $= G$. For any of the arrangements the mean constant γ_0 is given by the relation $\gamma_0 = G(1 - \alpha)$, where α is very small because the ratio of solenoid diameter to length is small, because the solenoid always extended far beyond the rotor ends,[15] and because the gaps between the upper and lower parts of each solenoid were small. For the different arrangements the quantitites α were carefully determined, and ranged from 0.0015 for a short or long rotor in the smaller solenoid to 0.0043 and 0.0078 for a short and long rotor in the larger solenoid.

The inductances of the two solenoids were approximately 6×10^{-5} henry and 17×10^{-5} henry.

[15] Except in a few early experiments.

§ 39. *The large Helmholtz coil.* The constant Γ_0 of the small compensating coil was determined electrically by comparison with the constant Γ_0' of a specially constructed Helmholtz coil. The same Helmholtz coil, together with a standard current, served to determine the intensity of the magnetic field by which the moment of the vibrator's magnet was found.

The diameter D of the Helmholtz coil was approximately 26 cm, and the axial distance A between centers of corresponding wires in the two halves was approximately 13 cm. Each half-coil contained 10 turns, placed close together, and was nearly 6 mm. wide.

The two half-coils were wound in accurate cylindrical grooves about 11 mm. wide turned in a round tube, 3.2 mm. thick, of bakelite micarta; see Fig. (39–1). The wires were of DCC copper No. 25 and were pressed tightly against one another and the outer shoulders of the grooves. They were held in position by tightly wound fish-cord filling the rest of the grooves and bound to them with shellac along the inner edges.

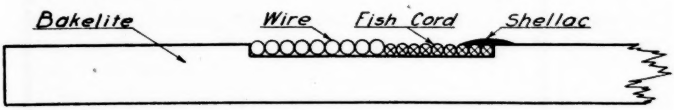

FIG. 39–1.

The end corrections and lead corrections were negligible. The axial distances A were measured with a dividing engine kindly loaned by Mr. Babcock of the Mt. Wilson Observatory. The mean value from several hundred properly spaced measurements is 13.170 cm and is correct to a degree of precision much beyond the requirements of this work.

The overall diameters were measured with calipers and a steel scale by Brown and Sharpe; also with a steel tape compared with the same scale. Both sets of measurements (taken over two years apart) agreed closely, but the latter were more precise. The tape measurements, properly corrected for tape thickness and wire diameter, gave for the mean diameter $D = 25.93_3$ cm. The diameters of the two half-coils differed by only about 1 part in 4000.

The constant of the coil is thus

$$\Gamma_0' = \frac{2\pi \times 20 \times (12.966)^2}{\{(12.966)^2 + (6.585)^2\}^{\frac{3}{2}}} = 6.869 \text{ e.m.u.} \qquad (39\text{–}2)$$

While in use, this coil was mounted with its axis horizontal in a brass frame as indicated in Figure 39–2, the frame being mounted on a brass tripod.

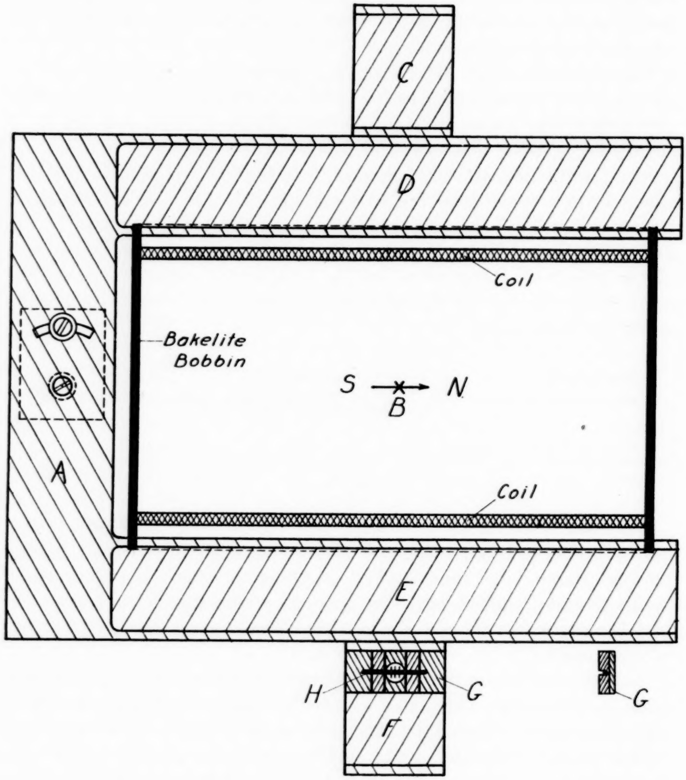

FIG. 39–2.

§ 40. *The constant of the torque coil.* For making the comparisons between the constant Γ_0 of this small coil and that of the larger Helmholtz coil, a special magnetometer and stand were constructed. When the small coil was removed from the vibration apparatus and

set properly on this stand, and the stand leveled and adjusted in height, the magnetometer magnet, almost exactly similar in construction to that on the vibrator, was at the center of all the coil systems, and their turns were parallel to one another.

In making the comparisons each coil, as is usual, was connected in series with a suitable rheostat; the two systems were then placed in parallel and connected through a reversing switch with the terminals of a storage battery. One of the rheostats was adjusted until the magnetometer needle gave a very much smaller deflection on reversal of the currents than that due to either coil separately, and then interpolation was made for zero deflection. The constants of the two coils were then proportional to the resistances of the respective branches.

In this way, in February, 1929, long after earlier less precise measurements, Γ_0/Γ_0' was found to be 266.52, corresponding to $\Gamma_0 = 1830.7$ e.m.u. A later determination, made in November and December, 1930, gave $\Gamma_0/\Gamma_0' = 266.72$ and $\Gamma_0 = 1832.0$. The difference between the results of the two measurements is 0.07 per cent, which is much beyond the possible experimental error. It cannot be attributed to a change in Γ_0', as control observations on the latter several years apart have shown.

The smaller value of Γ_0 has been used in the calculations for all observations before 1930; the larger for those of 1930.

It was originally planned to use the intermediate Helmholtz coil in every series of observations in order to redetermine the constant Γ_0 for the actual position of the vibrator magnet, and also in order to produce deflections of the vibrator magnet from which its moment, once determined absolutely, might be readily found as a function of the time. Experience, however, proved that the small errors made in setting the vibration system produced negligible errors[16] in the constant of the smaller coil; and it turned out to be more convenient to use the magnetometer for the necessary moment determinations than to use the Helmholtz coil.

§ 41. *The constants of the principal magnetizing coils.* The constants of the solenoids wound on the rotors were approximately 940 and 710

[16] Thus an axial displacement of the magnet from the center by about 1.25 mm. changed the constant by less than 1 part in 3000 and a lateral or vertical displacement of 1 mm. changed the constant by about 1 part in 3000; while in the vibration work the adjustment was in general correct to a fraction of a millimeter.

e.m.u. per layer for the No. 40 and No. 36 wire, respectively, and for points at the centers. The constants at the centers of the fixed magnetizing coils with 46 turns per inch per layer were approximately 270 e.m.u. per layer.

§ 42. *The moments of the vibrator magnets.* These were determined at suitable intervals with the aid of a simple magnetometer and the large Helmholtz coil.

The magnet of the magnetometer was one of the circular gages of polished steel, 1 mm. thick and 20 mm. in diameter, made at the Bureau of Standards, magnetized along a horizontal diameter, and suspended by a long strip of phosphor-bronze. One of the polished surfaces served as a mirror, for reading deflections by lamp and scale. The magnetometer box was of copper, and provided adequate damping. The scale distance was nearly 2 m.

The Helmholtz coil, carried by its bakelite bobbin, was mounted on its brass frame A, Fig. 39–2, with its center coincident with that of the steel magnet B and its axis horizontal and normal to the mirror, in the magnetic prime vertical.

The horizontal frame A which carried the coil had parallel grooves C, D, E, F milled in it symmetrically on both sides the coil and perpendicular to its axis. Small carriers G, two of brass and one of translucent bakelite, each with rectangular base and sides normal thereto, were accurately constructed to hold the vibrator magnets H symmetrically with the magnetic axes horizontal and normal to the greater lengths. The width of each magnet-carrier, and the distances between corresponding groove-edges on opposite sides of the magnetometer, were accurately measured. The groove-edge distance principally used was measured with the engine mentioned above. The carrier could be so set, by pressing it to the edge of a groove and moving it along this edge, that the axis of the vibrator magnet pointed to the center of the magnetometer magnet.

Since in the course of a determination of moment the carrier was placed the same number of times on each (east and west) side of the magnetometer; and since the magnet, fixed in the carrier, was turned east and west the same number of times; twice the mean distance between the centers of the two magnets was equal to the distance between the groove edges plus or minus the width of the magnet carrier, according as inner or outer groove edges were used.

In nearly all of the work the brass carrier No. 2, 1.93_4 cm wide,

was placed against the inner edges of two grooves distant 25.96_8 cms apart. Thus the mean magnet distance was 13.95 cm.

Let D denote the mean double deflection produced by reversing a standard current C in the Helmholtz coil, d the mean double deflection produced by reversing the axis of the vibrator magnet, m_0 the moment of this magnet, and r the distance between the centers of the two magnets. Then, if we can neglect the linear dimensions of the magnets, and assume proportionality of deflections and intensities,

$$\frac{d}{D} = 2m_0/r^3 \div \Gamma_0'C \quad \text{or} \quad m_0 = \frac{\Gamma_0'Cr^3}{2} \cdot \frac{d}{D}$$

The finite length of the vibrator magnets, viz. 4.2 mm. (= 5 mm. × 5/6), necessitates a negative correction − 0.18 per cent to m_0 as given by the above formula. The finite diameter of the magnetized mirror, viz. 20 mm., on the assumption that it is uniformly magnetized parallel to the horizontal diameter with poles distributed around the circumference, necessitates two corrections to m_0: one, equal to + 0.76 per cent, on account of the fact that the lines of induction from the vibrator magnet make (at the edge of the disc) an angle with the disc's axis equal approximately to $3 \times \frac{1}{2} \times {}^1/_{14}$ radian; and the other, equal to + 0.77 per cent, because the distance from the center of the vibrator magnet to the edge of the disc is greater by about 0.036 cm than the distance between the centers of the two magnets.[17]

The deflections are not, as above assumed, strictly proportional to intensities, and d/D must be multiplied by the correction factor 1.0040, or increased by 0.4 per cent, as careful experiments have shown.

The total correction is thus (+ 0.77 + 0.76 − 0.18 + 0.40) per cent = + 1.75 per cent. The true[18] value of m_0 is thus given, with negli-

[17] The correction whose theoretical value on the approximate assumption made is 0.76% + 0.77% = 1.53% was also determined experimentally with great care. The experimental value is 1.56%. Thus the difference between the correct value, which must be somewhat *less* than the theoretical value, and this theoretical value is negligible.

[18] When the observations are being made, the magnetometer magnet, whose moment is about 17 e.m.u., is always turned in such a way as to produce a magnetic intensity at the position of the vibrator magnet whose component h parallel to the axis of this magnet is in the direction of the axis and thus tends to increase the moment. The magnitude of h, however, for the maximum deflections was only 1.4×10^{-4} e.m.u., so that its effect on m_0 was entirely negligible.

Also, if θ denotes the angle between the axis of the magnet system and the

gible error, by the equation

$$m_0 = \frac{\Gamma_0' C r^3}{2} \frac{d}{D} (1 + 0.017_5) \text{ e.m.u.} \tag{42-1}$$

The current C was determined by means of a Wolff 15,000 ohm potentiometer, a Weston cell, and a standard coil with resistance 9.994 ohms, all properly checked by comparison with other standards and known to introduce only negligible errors.

A representative example of one determination of the moment of the vibrator magnet-system mostly used (No. 3) follows:

A. Time of beginning: 1^h 30^m a. m. June 27, 1928.

B. Current $C_1 = 0.004209_5$ ampere.
Deflection $D_{12} = 43.51 \pm 0.03$ cm. (from 18 scale readings at equal intervals).
Current $C_2 = 0.004209_5$ ampere.

C. Magnetometer readings and deflections I (from 32 readings at equal intervals):

N pole E		N pole W		N pole W		N pole E	
(1)	− 4.51 cm	(2)	+ 4.23 cm	(3)	+ 4.08 cm	(4)	− 4.40 cm
(8)	.47	(7)	.21	(6)	.07	(5)	.40
(9)	.48	(10)	.27	(11)	.10	(12)	.37
(16)	.43	(15)	.30	(14)	.11	(13)	.37
(17)	.46	(18)	.30	(19)	.12	(20)	.32
(24)	.40	(23)	.33	(22)	.15	(21)	.35
(25)	.41	(26)	.32	(27)	.18	(28)	.28
(32)	.40	(31)	.35	(30)	.17	(29)	.30

From (1) − (8) and (25) − (32), $d = 8.60$ cm
From (9) − (24), $d = 8.60$ cm
$\Big\}\ d_1 = 8.60$ cm

---

horizontal when the moment is measured, the quantity obtained is $m_0 \cos \theta$ instead of m_0. An error as great as 2° is setting the magnet, however, would affect the measurement by not much more than 1/20%, so that the error made by assuming $\theta = 0$ is negligible. Such error as there is makes the estimated values of m_0 and ρ too small.

When the magnet is in position at the center of the torque coil, the angle θ' between its axis and the planes of the turns of the coil was, in all the later work, made so small that the difference between $m_0 \cos \theta'$, to which the torque is proportional, and m_0 was negligible. Slight errors from this source, making the estimated value of the quantity ρ too large, occurred in some of the earlier and less precise work, and may account for a part of the small difference between the earlier and later results.

D. Current $C_3 = 0.004209_5$ ampere.

Deflection $D_{34} = 43.47 \pm 0.01$ cm (from 26 scale readings at equal intervals).

Current $C_4 = 0.004209_0$ ampere.

E. Magnetometer readings and deflections II:

Thirty-two observations similar to those of C. From (1)–(8) and (25)–(32), $d = 8.61$ cm. From (9)–(24), $d = 8.62$ cm, Mean $d_{II} =$ 8.615 cm.

F. Current $C_5 = 0.004209_0$ ampere.

Deflection $d_{56} = 43.47 \pm 0.02$ cm (from 20 scale readings at equal intervals).

Current $C_6 = 0.004209_2$ ampere.

G. Time of ending: $2^h\ 50^m$, a. m.

From B, C, and D, $m_{0I} = 0.777_0 \times 1.0175$ e.m.u.

From D, E, and F, $m_{0II} = 0.778_2 \times 1.0175$ e.m.u.

Mean $m_0 = 0.777_6 \times 1.0175$ e.m.u.

The moment of each magnet-system was plotted as a function of the time, and the value needed for any determination of the gyromagnetic ratio was taken from the appropriate curve. The average departure of the observed values from the curves was not greater than about 1 part in 1000.

§ 43. *Neutralization of the earth's magnetic field.* The total intensity of the earth's field in the region occupied by the rotor was very nearly annulled by passing steady electric currents through three compensating coils.

One coil, A, compensated the northerly component X; another, B, the easterly component Y; the third, C, the vertical component Z.

A and B are what might be called oblong Helmholtz coils, designed to produce very nearly uniform fields in a central cylindrical region. In each system are two similar elongated approximately rectangular coils so mounted in parallel planes that the wires form approximately edges of a rectangular parallelepiped. Each system is about 2 meters long.

Each coil of A has five turns of heavily insulated wire, the longer sides being about 36 cm apart, and the planes of the two coils being about 21½ cm apart. Each coil of B has only one turn of bare wire, the longer sides being about 23 cm apart and the planes of the two being about 13 cm apart. Each coil pair produces in the central region an intensity parallel to the shortest sides of its parallelepiped.

The coils were mounted in a bakelite frame, with brass distance

pieces at the ends to keep the horizontal dimensions of A constant, and wood distance pieces for B. The coils were always kept stretched tight, the wires of B being mounted with permanent tension, those of A being stretched by a spring device at the top of the frame.

On account of the large amount of iron used in the construction of the laboratory, the magnetic field about the rotor was not strictly uniform. Careful measurements of the component X, however, showed that it increased upward linearly by only about $\frac{1}{2}$ per cent in the length of a rotor. On this account the wires in each coil of A were not left strictly parallel, but were set closer together at the top by nearly $\frac{1}{2}$ cm for each meter of length. The field produced by A itself over the length of a rotor, when its wires were parallel, was uniform to 1 part in 1800.

The uniformity of Y was not investigated; but Y was only a small part of X, viz. about one twelfth.

System C is a pair of horizontal square coils 1.75 m. on a side, mounted with their wires parallel and coaxial one below the other at a mean distance of about 0.9 m., and traversed by currents in series in the same direction. Each coil has 45 turns of DCC No. 20 copper wire. These square coils are easier to build than true (circular) Helmholtz coils, and produced an axial field nearly enough uniform. Indeed, the variation of their axial intensity from the center of one of the larger rotors to either end is only about 1 part in 600. The degree of uniformity of Z was not investigated, as only comparatively rough vertical compensation was necessary. The coil system C was hung from the ceiling by heavy brass wires attached below to brass slides permitting vertical adjustment, with set screws, and making it easy to level the coils with exactness.

The currents traversing A, B, and C were in general furnished by storage batteries used at the same time for no other purpose, and were measured at first with ammeters; later the potentiometer and adjunct ammeters were used for A and B.

§ 44. *Standardization of the compensating coils.* The proper currents to compensate X and Y at various times were determined by means of a special flip coil about an inch in diameter and as long as a rotor, the currents being adjusted until the flips produced a small deflection with a sufficiently sensitive galvanometer, and the currents for zero deflection being found by interpolation. The X and Y compensating currents were determined to about 1 part in 1000 and 1 part in 100 respectively. The currents to compensate the vertical component Z was determined in much the same way, an earth inductor being used

as a flip coil. The Z compensating current was set to about 1 part in 900, but the calibration was not so precise as this.

Determination of compensating currents at time of principal experiments. It was, of course, impossible to make the standardizing observations except at infrequent intervals. Hence, as the earth's magnetic field is continually fluctuating, it was necessary to make in connection with them other observations which would render it possible to determine the proper compensating currents at any time at which the principal experiments were in progress.

The following procedure was adopted: Before and after the flip coil determinations for X and Y, magnetometer scale readings were taken for a known current reversed through the standard Helmholtz coil; and similar magnetometer and current readings were taken at the times of the principal experiments.

Let X_0 and Y_0 denote the correct values of the compensating currents, D_0 the mean deflection of the magnetometer (without control magnets) produced by the current C_0, θ_0 the magnetic declination, and R_0 the zero reading on the magnetometer scale, all at the mean time T_0 of the standardization; and let X, Y, D, C, θ, and R denote the corresponding quantities at any other time T at which principal experiments are to be made. Then, very approximately,

$$X = X_0 \frac{C}{C_0} \cdot \frac{D_0}{D} \tag{44-1}$$

and

$$Y = Y_0\left(1 + \frac{X - X_0}{X_0} + \frac{\theta - \theta_0}{\theta_0}\right) = Y_0\left\{\frac{C}{C_0}\frac{D_0}{D} + 6\,\frac{(R - R_0)}{d}\right\} \tag{44-2}$$

since $\theta_0 = 1/12$ radian, and $\theta - \theta_0 = (R - R_0)/2d$, where d is the scale distance.

Similarly, an earth inductor is set up near the magnetometer in a permanent position, and at the time of calibration is reversed through a galvanometer, first in such a way as to cut only the horizontal lines of the earth's field, giving the deflection D_{H0}; second, in such a way as to cut only the vertical lines, giving the deflection D_{Z0}. Similar observations at the time T give deflection D_H and D_Z. Then, very approximately, if Z_0 denotes the vertical intensity at the center of the coils at the time T_0, and Z the intensity at the time T,

$$Z = \frac{Z_0}{X_0}\frac{D_{H_0}}{D_{Z_0}}\frac{D_Z}{D_H} \cdot X = Z_0\frac{D_{H_0}}{D_{Z_0}} \cdot \frac{D_Z}{D_H} \cdot \frac{C}{C_0}\frac{D_0}{D} \tag{44-3}$$

From (44–1), (44–2), (44–3), X, Y, and Z can be found at any time T.

45. *Observations and results.* By the methods now described a large amount of work has been done on rotors of permalloy and soft iron. They are illustrated by examples below and summarized in six groups, designated as Groups O, I, II, III, IV, and V. Group O contains the observations made by the null-graphical method, the others those made by deflections. The latter are also referred to as Deflections I, II, III, IV, V.

Groups O–III were made without any attempt to eliminate by reversals the effect of any inequality of the half-cycles of current. In the course of these observations, however, the magnetizing coils were removed from and connected into the circuit a great number of times with no attention whatever as to which wire from the generator or commutator was connected to a given terminal. The connections were thus probably reversed many times in the course of the work, and the error involved largely eliminated. That this is the case is indicated by the close agreement between the means of the results obtained. In much of this work the effects of the inequality were small, but they became prominent in a part of Group III, and led to the discovery of the source of the trouble.

Among the magnetostrictive effects alluded to in § 9, and in addition to that treated in § 17, is a small ripple, usually of (relatively) high frequency, superposed on the fundamental vibration. It appears to be a forced vibration due to axial asymmetry of the suspended system and the magnetostrictive elongations produced by one or more of the harmonics of the magnetization of the rotor. The ripple frequencies are thus twice the frequencies of these harmonics. When the ripple exists, the horizontal band of light on the scale is crossed by vertical shaded bands, or lines if the amplitude is small. When the rotor is unsuccessfully mounted, or the asymmetry otherwise too great, the appearance is not steady, and precise observations cannot be made. When the rotor is well mounted, but still without perfect symmetry, the appearance is very steady. The band of light in this case, when the amplitude is not too large, resembles somewhat a line spectrum with a continuous background.

This magnetostrictive ripple was present in many of the earlier observations; but it was judged to be absent in all or nearly all of those in Group V. Its presence, however, does not produce any error in the calculated value of the gyromagnetic ratio, either in the null-

graphical method, or in the deflection methods of Groups II–V when the amplitudes A_1 (or A) and A_2 are equal. When they are only approximately equal (as is usually the case) the error is of the second order and negligible. The vanishing of the error is due to the fact that the ripple affects amplitudes on both sides the minimum (when equal) exactly alike. In the method of Group I, where only the amplitude A_G can contain the ripple, the calculated value of ρ is slightly too large if the ripple is present.

The earliest observations were made in Group O, when the details of the procedure necessary to secure the best operation of the apparatus were imperfectly understood. A number of sets of observations were rejected because of a high minimum (probably due largely to the unsuspected effect of half-cycle inequality) in one of the two halves of the set, or in both halves, or because the compensating currents necessary to produce low minima differed by more than very small amounts. In almost all cases throughout the investigation the currents were set and maintained practically constant throughout the complete set; while in successive sets the first half was made with the axis of the vibrator magnet pointing alternately east and west, in order to correct, on the whole, for any slow fluctuation in the earth's field.

Most of the work described in this article has had to be done at or near the time of the sun-spot maximum. The work has therefore been greatly interfered with by numerous, and sometimes violent, magnetic storms, which have made work impossible on a large number of nights.

§ 46. *Observations for ρ by the null-graphical method.* (Group O). In the practise of this method, the earth's field is neutralized (or approximately neutralized), and the vibrating system set into resonance (or approximate resonance) so that a large amplitude is obtained with the induction circuit open; then the circuit is closed and the resistance adjusted until the amplitude is cut down to a few cm at most. Then the amplitude is determined for a number of different resistances, first decreasing until after the minimum is well past, then increasing until the minimum is well past again. A curve (or the reduced straight line) plotted from all the observations near the minimum gives the resistance (or the conductance) for the minimum approximately independently of the changes in conditions during the process. The observations are then repeated with the torsion head turned through 180°, and the connection between induction solenoid and compensating coil reversed. In order better to eliminate

errors due to changes in the earth's field, these pairs of observation groups on successive days are made in opposite orders.

Examples of curves obtained in this manner are given in Fig. 46–1. These curves were obtained in October 1927, with the rotor ${}_{8}P_{2\cdot4-2}$ mounted in the 1¼″ induction solenoid, a current of 100 milliamperes (rectangular wave) traversing the coil wound on the rotor. The moment of the vibrator magnet when these results were obtained was 0.7860×1.0175. From this, the resistances or conductances for minimum amplitude given by the charts, the coil constants given in the text, and the correction $-$ 0.002 for coil moment and electron inertia, the value of ρ/ρ_L for $N\ P\ E$ is 1.054, that for $N\ P\ W$ 1.065; mean 1.060. The observations from which Fig. 46–1 was drawn were obtained under poor conditions when the apparatus was constantly swinging. Curves of the same kind obtained now and under good conditions would be much better still.

All the observations between May 21, 1927, when the first good results were obtained, and February 23, 1928 were made by this general method, although details were not always the same. In this interval 16 sets[19] gave for iron, with the current traversing a coil wound on the rotor, $\rho/\rho_L = 1.056 + 0.011$; while 2 sets in which the magnetizing coil was fixed to the earth and the iron rotor had no winding gave 1.037 ± 0.002. The mean for the iron rotors is 1.054 ± 0.010.

In this same interval 21[19] sets on permalloy rotors with windings traversed by the current gave $\rho/\rho_L = 1.063 \pm 0.017$, while 9 sets in which the magnetizing coil was fixed gave $\rho/\rho_L = 1.069 \pm 0.005$. In 4 of these latter the rotor was the same as in the 21 sets just mentioned; in 5 a rotor without winding was used.

The rotors used were as follows: ${}_{36}I^{1}{}_{2\cdot4-2}$, $I^{2}{}_{2\cdot4-2}$, ${}_{8}I_{1\cdot5-0}$; ${}_{36}P^{1}{}_{2-2}$, ${}_{36}P^{2}{}_{2-2}$, ${}_{8}P_{1\cdot5-0}$, ${}_{8}P_{2\cdot4-2}$, $P_{3\cdot2-2}$, $P_{3\cdot2-4}$. Of the 21 sets first mentioned all but 6 were made on rotors from 2 to 2.4 mm. in diameter. The remaining six were made on rotors 3.2 mm. in diameter. This group of six gave $\rho/\rho_L = 1.063 \pm 0.020$.

The mean for the permalloy rotors is 1.064 ± 0.013.

The strength of the magnetizing field in all these experiments except one was in the neighborhood of 20 gausses (uncorrected for end effects). In the exceptional case (one of the group of 6) it was about 40 gausses, but the result differed from those with the weaker field by less than they differed among themselves.

In 10 of the sets on iron and 17 of the sets on permalloy the magnet-

[19] In two of these sets the remote induction solenoid was used (§ 27).

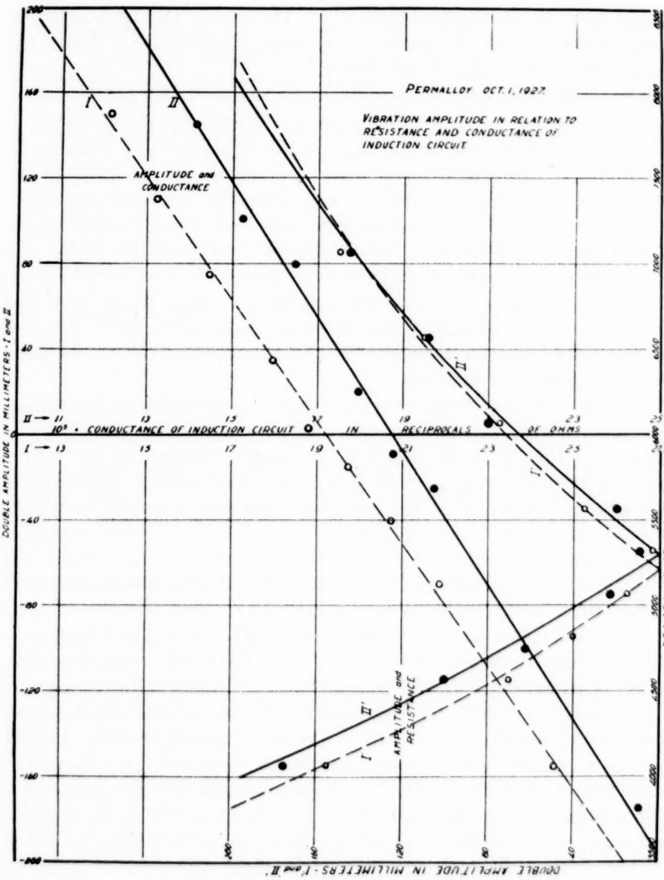

Fig. 46–1.

mirror system No. 1 was used. Long after the experiments were over the brass carriage in which the system was held when its moment was determined was found to be magnetized. If it was magnetized, and to the same amount, while in use, ρ/ρ_L for these 27 observations

would have to be *increased* by 0.8%. The earlier and later observations, however, do not disclose any certain difference. When the carrier was magnetized is not known.

The frequencies ranged from about 4.1 to about 13.9 per second.

§ 47. *Example of observations for* ρ. *Large deflection method I.* (Group II). For reasons given in § 20 comparatively precise observations by this method were restricted to a single set, although the method is probably superior to any deflection method previously used by others. The following data, most of which, including the gyromagnetic deflections, were obtained in December, 1928, will suffice to illustrate the practice of the method.

The rotor was ${}_{s}P_{2\cdot4-2}$, and the experiments were made with the complete rotor in a magnetizing coil fixed to the earth, the coil wound on the rotor being on open circuit. The strength of the magnetizing field was about 20 gausses.

During each set of observations, made at approximate resonance, the fork control was not altered, and the amplitudes gradually decreased. The effect of this was practically eliminated by taking the observations in the order indicated. The natural frequency of the rotor as mounted at the time indicated was $\nu = 6.84$ cycles per second, giving $\omega = 2\pi\nu = 43.9_7$. The value of m_0 was $0.768_6 \times 1.0175$, and the value of $Q \times \pi/4$ was 1.248×10^{-6} e.m.u. The value of Γ_0 was 1830.7 gauss/e.m.u. current. The additional data necessary are given in Table 47–1.

These data, inserted in equation (20–3) give for $|\rho|$ the values $\rho_1 = 10^{-7} \times 0.5967$, and $\rho_2 = 10^{-7} \times 0.5947$. The mean, from which the effect of disturbing torques, except the possible effects due to inequality of the half-cycles and magnetostriction, apparently small at this time, are eliminated, is $|\rho| = 10^{-7} \times 0.5957$. This gives $\rho/\rho_L = 1.054$.

§ 48. *Observations by the second deflection method.* (*A*) *Large and small deflections.* Group II. The procedure and general character of the results are best illustrated by a table such as Table 48–1, which contains observations of March 22, 1928, made according to § § 25 and 26 with both small and large deflections. The observations by this method were in general made in the definite order indicated, and usually on a regular time schedule so as to eliminate errors arising from the inevitable changes in conditions during the progress of the work. In the example given the magnetizing current traversed the solenoid wound on the rotor. These observations also were obtained on a relatively poor night, as the building was in a state of continual

TABLE 47-1.

Group	Azimuth of vibrator magnet	Order of Obs.	Magnetizing current	$\pi/4 \times M$ (e.m.u.)	A^*_G	A^*_C
1	North pole west	(1)	337 milli-amperes	627	8.50 cm	— cm
		(2)	—	—	—	9.25
		(3)	—	—	—	9.05
		(4)	337	627	8.35	—
	Mean amplitudes				8.425	9.15
2	North pole east	(1)	335	626	8.40	—
		(2)	—	—	—	9.20
		(3)	—	—	—	8.70
		(4)	335	626	8.00	—
	Mean amplitudes				8.20	8.95

* Minimum amplitudes with gyromagnetic torque balanced were not obtained on this occasion, but observations made shortly before and after make it evident that the quadrature torques had a negligible effect on A_G and A_C.

vibration; nevertheless those obtained with the different arrangements agree remarkably well. The residual amplitude was not measured, but was unnecessary, as the observations show.

Seventy sets of observations, complete except for systematic reversals to eliminate the effect of possible inequality of the half-cycles, were obtained in the interval Feb. 1928–Jan. 1929 by this method on four rotors of permalloy and soft iron, viz., $I^2_{2\cdot4-2}$, ${}_sI_{1\cdot5-0}$, ${}_sP_{2\cdot4-2}$, and ${}_sP_{1\cdot5-0}$; and the results are given in Table 48–2. The columns (1), (2), (3) correspond to columns 3, 4, and 2 of Table 48–1, respectively. On account of the possible error due to residual minimum amplitude, the results of columns (1) and (3) are more reliable than those of column (2).

§ 49. *Observations by the second large deflection method, con.* (*B*). *Large deflections only.* (Group III). The practice of observing with both small and large amplitude was later abandoned, and observations made with the large amplitudes only, the number of large amplitudes observed in one set being increased (ordinarily) to 12. Table 49–1 gives in tabular form most of the results obtained prior

TABLE 48–1.

Observations for Gyromagnetic Ratio, March 22, 1928.

Rotor: $I^2_{2.4-2}$, Current: 100 milliamperes (field strength 20 gausses). Frequency: 6.7 per second. $m_0 = 0.779_6 \times 1.0175$ e.m.u.

I. Azimuth of vibrator magnet: N pole west. Times of beginning and ending: $2^h\ 25^m$ and $3^h\ 40^m$ a. m.

Resistance of induction circuit in ohms	∞	2755	4655	6055	5055	5555
Amplitudes × 2 in cms	*(1) 15.90	(2) 15.00	(15) 2.50	(16) 1.90	(3) 1.10	(4) 0.90
	(28) 15.70	(27) 14.60	(18) 2.45	(17) 1.90	(6) 1.45	(5) 0.84
	*The numbers in parentheses indicate the order of the observations		(19) 2.50	(20) 1.90	(7) 1.50	(8) 0.92
			(22) 2.45	(21) 1.90	(10) 1.18	(9) 0.90
			(23) 2.38	(24) 2.00	(11) 1.18	(12) 0.93
			(26) 2.30	(25) 1.95	(14) 1.10	(13) 0.91
Mean amplitudes × 2	15.80	14.80	2.43	1.92_5	1.25	0.90

II. Azimuth of vibrator magnet: N pole east. Times of beginning and ending, $4^h\ 15^m$ and $5^h\ 35^m$ a. m.

Resistance of induction circuit in ohms	∞	2755	4655	6055	5055	5555
Amplitudes × 2 in cms	(1) 15.46	(2) 14.50	(3) 2.40	(4) 2.05	(15) 1.50	(16) 1.26
	(28) 15.70	(27) 14.50	(6) 2.55	(5) 2.08	(18) 1.37	(17) 1.28
			(7) 2.55	(8) 2.10	(19) 1.36	(20) 1.11
			(10) 2.55	(9) 2.10	(22) 1.30	(21) 1.10
			(11) 2.50	(12) 2.00	(23) 1.30	(24) 1.40
			(14) 2.58	(13) 2.00	(26) 1.55	(25) 1.35
Mean amplitudes × 2	15.58	14.50	2.52	2.05_5	1.40	1.25

*ρ/ρ_L from I	1.043	1.041	1.044
*ρ/ρ_L from II	1.049	1.046	1.049
Mean*ρ/ρ_L	1.046	1.044	1.047
Mean†ρ/ρ_L	1.044	1.042	1.045

* Uncorrected for coil moment and electron inertia.

† Corrected for coil moment and electron inertia.

TABLE 48–2.

ρ/ρ_L FROM SMALL (1), SMALLER (2), AND LARGE (3) AMPLITUDES (GROUP II).

Rotor	Approximate field strength in gausses	Frequency in cycles per sec.	No. of sets	(1)	(2)	(3)
A. Iron						
$I^2_{2\cdot4-2}$	2	6.6	1	1.030	1.030	1.016
$I^2_{2\cdot4-2}$	5	6.6	6	1.038 ± 0.007	1.044 ± 0.005	1.032 ± 0.010
$I^2_{2\cdot4-2}$	10	6.6	14	1.038 ± 0.005	1.040 ± 0.005	1.036 ± 0.005
$I^2_{2\cdot4-2}$	20	6.6 & 8.6	18	1.046 ± 0.007	1.046 ± 0.004	1.046 ± 0.009
${}_sI_{1\cdot5-0}$	20	8.1	2*	1.046 ± 0.008	1.048 ± 0.003	1.026 ± 0.014
	Mean		41	1.042 ± 0.006	1.043 ± 0.004	1.039 ± 0.008
B. Permalloy						
${}_sP_{2\cdot4-2}$	5	6.9	8	1.038 ± 0.008	1.042 ± 0.009	1.041 ± 0.007
${}_sP_{2\cdot4-2}$ & ${}_sP_{1\cdot5-0}$	5	6.9 & 7.6 ±	6*	1.042 ± 0.008	1.053 ± 0.006	1.023 ± 0.008
${}_sP_{2\cdot4-2}$	10	6.9	3	1.049 ± 0.002	1.048 ± 0.001	1.055 ± 0.004
${}_sP_{1\cdot50}$	10	7.4 & 7.7	4*	1.058 ± 0.005	1.061 ± 0.002	1.056 ± 0.003
${}_sP_{2\cdot4-2}$ & ${}_sP_{1\cdot5-0}$	20	6.9 & 7.6 ±	11†	1.047 ± 0.006	1.047 ± 0.005	1.043 ± 0.005
	Mean		32	1.045 ± 0.007	1.049 ± 0.005	1.041 ± 0.006

* With magnetizing coil fixed to earth.

† Includes 3 sets with coil fixed to earth.

to August 8, 1929, when further modifications were made in the method. Some of the observations were obtained under extreme conditions, and are quite rough. This is especially true of Groups 1 and 2, and in particular of Group 1, in which the heat dissipated in the winding of the rotor produced great irregularities in the vibrations. Several sets of observations made with a rotor in the 80 gauss field of a fixed magnetizing coil gave a considerably lower value of ρ and are not included in the table. The low value is probably due in part to the non-verticality of the axis of the magnetizing coil, and its different altitudes in the two half-sets (the error introduced being much greater in strong than in weak fields); and in part to the inequality of the half-cycles of current, the effects of which were now becoming manifest. Groups 3 and 7–8 are far more reliable than Groups 1–2 and 4–6 and alone of these observations are included in the final summary given in the next section.

§ 50. *Observations by the second large deflection method, con. (C). Observations in which the effect of inequality of half-cycles was approximately eliminated in each half set.* Group IV. The observations were in general similar to those recorded above, except that reversals were systematically made in each set to eliminate the effect of magnetostriction. The results of all but the final group of observations are given in Table 50–1.

§ 51. *Observations by the second large deflection method, con. (D) Final observations on permalloy and soft iron.* Group V. For these observations a more rigorous time schedule was adopted, and the observations were taken in such an order that the mean times for both large amplitudes in each half set (one for each azimuth of the vibrator magnet) were very nearly identical. An example is given in Table 51–1.

The results of all the observations made in this way, with the exception of one set on an iron rotor in a weak field, which was evidently vitiated by some unknown cause, and which gives a very large value of ρ/ρ_L, are summarized in Table 51–2.

[*Addition made in proof:* Since this paper went to press, and with the help of Dr. Otto Beeck, ten sets of observations have been made by this method on the large permalloy rotor $P^n{}_{3\cdot 2-4}$ with a field strength of about 35 gausses and at a frequency of about 19 cycles per second. The observations are not so reliable as those of Group V, because the vibrator magnet system had to be reconstructed and remagnetized before the observations began, and because while the series was in

TABLE 49-1.

ρ/ρ_L From Large Amplitudes (Group III).

Material	Group	Rotor	Field strength in gausses	Frequency in cycles per sec.	No. of sets	Range of ρ/ρ_L	Mean ρ/ρ_L
Iron	1*	$I_{2.4-8}$	80	17.4	6	1.027 – 1.158	1.111 ± 0.045
	2	$I_{2.4-8}$	20	17.4	6	0.901 – 1.072	1.022 ± 0.024
	3	$I^2_{2.4-2}$	20	8.1	3	1.026 – 1.051	1.042 ± 0.011
Permalloy	4	$P_{2.0-8}$	40	19.7	5	1.003 – 1.052	1.023 ± 0.015
	5	$P_{2.0-8}$	80	19.7 & 18.0	5	1.026 – 1.042	1.035 ± 0.007
	6†	$_sP_{1.5-0}$	20	19.7	7	1.040 – 1.076	1.064 ± 0.015
	7	$P^2_{3.2-4}$	40	5.5	6	1.035 – 1.059	1.047 ± 0.007
	8	$_sP_{2.4-2}$	20	6.9 & 7.7	6	1.028 – 1.058	1.048 ± 0.008

* For two of these sets curves were obtained as in the earlier work, and the calculations were made from them, giving $\rho/\rho_L = 1.043 \pm 0.016$. These are the most reliable observations of the group.

† Coil fixed to the earth.

TABLE 50–1.

MAGNETOSTRICTION APPROXIMATELY ELIMINATED IN EACH HALF-SET (GROUP IV).

Material	Group	Rotor	Field strength in gausses	Frequency in cycles per sec.	No. of sets	Range of ρ/ρ_L	Mean ρ/ρ_L
Permalloy	1	${}_sP_{2\cdot4-2}$	20	6.9	16	1.022 – 1.090	1.055 ± 0.016
	2	${}_sP_{2\cdot4-2}$	5	8.0	11	1.047 – 1.075	1.058 ± 0.008
	1 & 2	${}_sP_{2\cdot4-2}$			27		1.056 ± 0.013
Iron	3	$I^2_{2\cdot4-2}$	20	10.0	14	1.020 – 1.052	1.040 ± 0.008

TABLE 51-1. MARCH 18, 1930.

EFFECTS OF HALF-CYCLE INEQUALITY COMPLETELY ELIMINATED IN EACH HALF-SET.

ROTOR $I^2_{2.4-2}$. FREQUENCY 9.9 PER SEC. FIELD STRENGTH 15 GAUSSES. MOMENT OF VIBRATOR MAGNET: 1.0175 × 0.7954 E.M.U. TIMES OF BEGINNING AND ENDING: $1^h\ 45^m$ AND $5^h\ 46^m$ A. M.

Azimuth	North Pole West				North Pole East			
Position of switch	I	II	II	I	I	II	II	I
Box resistance in ohms¶	∞	2600	∞	2600	∞	2600	∞	2600
Amplitudes × 2 in cm	(1)* 11.40 (8) 11.48 (9) 11.55 (16) 11.68 (17) 11.78 (24) 11.40	(2) 11.35 (7) 11.76 (10) 11.63 (15) 11.93 (18) 11.83 (23) 11.55	(3) 9.56 (6) 9.58 (11) 9.75 (14) 9.83 (19) 9.87 (22) 10.33	(4) 10.11 (5) 10.13 (12) 10.10 (13) 10.17 (20) 10.25 (21) 10.33	(1) 11.83 (8) 11.73 (9) 11.60 (16) 11.53 (17) 11.73 (24) 11.83	(2) 12.15 (7) 12.35 (10) 12.05 (15) 12.17 (18) 12.45 (23) 12.43	(3) 10.43 (6) 10.16 (11) 10.38 (14) 10.49 (19) 10.73 (22) 10.51	(4) 10.93 (5) 11.03 (12) 10.73 (13) 10.98 (20) 11.33 (21) 11.23
Mean amplitude × 2 in cm	11.55	11.68	9.82	10.18	11.71	12.27	10.45	11.04
Mean residual amplitude × 2 in cm	0.9		0.9		0.8		0.5	
†ρ/ρ_L	1.125 (from I)		0.966 (from II)		1.089 (from I)		0.973 (from II)	
Mean† ρ/ρ_L	1.046 (NPW, I & II)				1.031 (NPE, I & II)			
Mean ρ/ρ_L‡ for both azimuths	1.037 (NPW & NPE)							

* The numbers in parentheses indicate the order of the observations for each azimuth.

† Uncorrected for coil moment and electron inertia.

‡ Corrected for coil moment and electron inertia.

¶ Extra resistance in circuit, 69 ohms.

progress a piece of one of the twelve magnets broke off, making somewhat uncertain extrapolations necessary to determine some of the moments. These observations, corrected for nearly one percent electron inertia effect, give $\rho/\rho_L = 1.046 \pm 0.005$, with the range 1.038 $-$1.054. The result is entirely consistent with the results on permalloy in Group V. The vibrations were entirely free from magnetostrictive ripple.]

§ 52. *Summary of results.* The mean results obtained by the different methods are summarized in Table 52-1. There is but little disagreement between them. The results from the observations of Group V are far more trustworthy than the others, although these others are considered much more reliable than any obtained by earlier investigators. The new results are all entirely consistant with those published by the author and L. J. H. Barnett in 1925, which give as the mean value of ρ/ρ_L from experiments on magnetization by rotation the number 1.05_9. The differences are less than the sum of the probable experimental errors.

The work on magnetization by rotation, with a mean error between one and two per cent, was not sufficiently precise to reveal any probable differences between the gyromagnetic ratios for different substances. The results now obtained on rotation by magnetization, however, appear to show a distinct difference between permalloy and iron, the gyromagnetic ratio for the former being about one per cent greater than for the latter. This difference is considerably greater than the average departures from the means in the most reliable work.

The work has revealed no differences in the gyromagnetic ratio due to changes in dimensions of rotor, field strength, or frequency. It has also shown that the gyromagnetic torque acts upon the magnetic material of the rotor only, or at least that no appreciable part of it acts upon the magnetizing coil, which may be fixed either to the rotor or to the earth with no difference in result.

§ 53. *The error in ρ.* The error in ρ depends on the errors in Γ_0, γ_0, R_0, and m_0. The error in γ_0, and that in R_0 in so far as it depends on the correctness of the box resistances, may be considered entirely negligible. The error in Γ_0 is probably less than 1 part in 2000. The only error of any consequence arising from imperfect knowledge of the constants is that in m_0, which depends on Γ_0' C, r^3, and d/D. The error in each of the first three quantities is probably not greater than 1 part in 4 or 5 thousand, and that in d/D not as great as 1 part in 1000. Altogether, m_0 may be in error by about 0.2 per cent; but this is

TABLE 51-2.

RESULTS OF FINAL OBSERVATIONS ON PERMALLOY AND SOFT IRON (GROUP V).

Material	Group	Rotor	Field strength in gausses	Frequency in cycles per sec.	No. of sets	Range of ρ/ρ_L	Mean ρ/ρ_L
Permalloy	1	${}_sP_{2\cdot4-2}$	20	10.0	17	1.046 – 1.055	1.049 ± 0.003
Iron	2	$I^2_{2\cdot4-2}$	20	9.8	16	1.028 – 1.039	1.035 ± 0.002
	3	$I^2_{2\cdot4-2}$	30	9.9	4	1.033 – 1.046	1.039 ± 0.004
	4	$I^2_{2\cdot4-2}$	15	9.9	5	1.036 – 1.049	1.040 ± 0.004
Iron	2, 3 & 4	$I^2_{2\cdot4-2}$			25		1.037 ± 0.003

TABLE 52–1.

SUMMARY OF RESULTS OBTAINED FOR THE GYROMAGNETIC RATIO.

	Material	Iron		Permalloy	
Group	Method	No. of sets	ρ/ρ_L	No. of sets	ρ/ρ_L
0	Graphical-Null	18	1.054 + 0.010	30	1.064 ± 0.013
I	Deflection I			1	1.054
II*	Deflection II	41	†1.040 + 0.007	29	†1.043 ± 0.006+
III**	Deflection III	3	1.042 + 0.011	12	1.048 ± 0.008
IV	Deflection IV	14	1.040 + 0.008	27	1.056 ± 0.013
V	**Deflection V**	**25**	**1.037 + 0.003**	**17**	**1.049 ± 0.003**

* From columns (1) and (3) of Table 48–2.

** Groups 1 and 2 and 4–6 of Table 49–1 are excluded.

† Mean from columns (1) and (3) of Table 48–2.

likely an over-estimate. So far as the constants are concerned, the error in ρ is not likely greater than 0.2%, even if all the componens errors are in the same direction, which is improbable. And this it less than the average departure from the mean in the final work. It therefore seems exceedingly probable that the final values of ρ are not in error by as much as one-half per cent.

§ 54. *The magnetic elements.* The differences of 4 or 5 per cent between the chief results of this investigation (as well as the investigation by L. J. H. Barnett and myself on magnetization by rotation) and those of the British physicists who have worked on rotation by magnetization are, as stated above, probably due, at least largely, to the failure of these investigators to eliminate the effects of magnetostriction and other effects depending on inequality of half-cycles of current, or on residual magnetization. There is no longer any evidence for the exact equality of ρ and m/e.

What interpretation must finally be given to the difference which is now confirmed is not yet certain. Inasmuch, however, as a gyromagnetic ratio equal almost exactly to m/e is apparently demanded in order to explain the Zeemann effect and a great mass of other spectroscopic phenomena, it seems probable that the chief magnetic element in ferromagnetic substances is the spinning electron with $\rho = m/e$, but that there is some contribution due to electron orbital motion. If this is true, we must expect to find for any substance from experiment (and calculations based on the hypothesis of only one kind of element) a mean value of ρ somewhere between m/e and $2m/e$; and in particular the small apparent difference between the values of ρ for permalloy and iron is not surprising.

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